Risk and the Allocation of Talent in the Roy Model

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Abstract

With risk-averse workers and uninsurable earnings shocks, competitive markets allocate too few workers to jobs with high earnings uncertainty. Using an equilibrium Roy model with incomplete markets, we show that in competitive equilibrium, risky occupations are inefficiently small and hence talent is misallocated.

Key words: Misallocation, Occupations, Risk, Incomplete Markets, Roy Model.
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1 Introduction

Misallocation of talent across occupations lowers productivity. This paper is the first to study how incomplete markets shape the aggregate allocation of talent and aggregate output. Through the prism of a Roy model, we show that talent is misallocated in a *laissez faire* competitive equilibrium. Risk averse workers avoid risky occupations when insurance opportunities are absent, unless wages are sufficiently high. But at high wages the demand for workers is low and as a result risky occupations are inefficiently small. Therefore, output gains can be achieved by reallocating workers across occupations.

Our general equilibrium Roy model features a labor market where risk-averse workers self-select into an occupation based on their comparative and absolute advantages. Human capital (for example acquired through specialized training) is specific to an occupation. Workers' occupational choices determine the level of output in the economy. We compare the level of production in competitive equilibrium to the first best: the allocation obtained by an unconstrained planner which maximizes welfare.

1.1 Related Literature

Our theoretical approach uses the insights of Roy (1951) and models workers' occupational choice under uncertainty. Thus, it connects to models of occupational choice used in macroeconomics and labor economics. Examples include Kambourov and Manovskii (2008), Jovanovic (1979), Miller (1984) and Papageorgiou (2014). We model the interplay between skills and risk so we complement their findings as well as the ones of Cubas and Silos (2017), Hawkins and Mustre del Rio (2012), Dillon (2016), and Neumuller (2015). Our paper also relates to the macroeconomics literature on the misallocation of human capital (see for example Vollrath (2014) and Hsieh, Hurst, Jones, and Klenow (2019), Buera, Kaboski, and Shin (2011)).

2 Model

The economy is populated by a mass of size one of workers who live for one period. They are endowed with a unit of time which they inelastically supply as labor in either of two occupations (R for risky and S for safe). Workers value the consumption of a final good produced according to the following CES technology.

$$Y = [\theta N_R^{\nu} + (1-\theta) N_S^{\nu}]^{1/\nu} \tag{1}$$

where N_R and N_S are the aggregate amount of efficiency units of labor in the risky and safe occupations, respectively, $0 < \theta < 1$ governs the share of each occupation in total output and ν is the elasticity of substitution between the two occupations.

Workers finance final goods with labor earnings since they don't save and start with zero wealth at birth. In our context, career risk is permanent for workers, and their preferences are captured by a constant relative risk aversion utility function. Individuals ranks consumption levels c according to $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, with $\gamma > 1$.

Workers are endowed with a vector of occupation-specific abilities. Abilities can be correlated across occupations and as a result some workers are likely to excel at several professions. In what follows, the vector of abilities is denoted by $\boldsymbol{X} = (X_R, X_S)$. We model the dependence between the two abilities through a Gumbel copula of two Fréchet random variables:

$$F(x_R, x_S) = Pr(X_R < x_R, X_S < x_S) = \exp\left\{ -\left[\sum_{i \in R,S} (T_i^{\alpha} x_i^{-\alpha})^{1/(1-\rho)} \right]^{(1-\rho)} \right\} \tag{2}$$

The parameter T_i is the scale parameter. The parameter, $0 < \rho < 1$ controls the dependence across ability levels for a given worker. When ρ approaches 1 there is perfect dependence between the two ability draws. When it approaches zero, abilities are uncorrelated. The parameter α drives the dispersion and it is common to all

abilities. [1]. Given (2), the marginal distributions are standard univariate Fréchet with cdf

$$Pr(X_i < x_i) = \exp\left\{-\left(\frac{x_i}{T_i}\right)^{-\alpha}\right\} \tag{3}$$

We derive this result in Section A of the Online Appendix.

2.1 Occupational Choice and Sorting

Given a realization of $\boldsymbol{X} = (x_R, x_S)$, a worker chooses between two careers. In one, earnings are less predictable, with occupation R being riskier. Uncertainty arises from shocks affecting a worker's ability in each occupation, following distributions $F_i(y)$ for occupations $i = R, S$. We assume these shocks are log-normally distributed with a mean of one and a variance of $var(log(y_i)) = \sigma_i^2$. Occupational choice depends on predetermined abilities $\boldsymbol{X}$ but is independent of subsequent job-related shocks. To formalize the occupational decision given $\boldsymbol{X}$ and the market prices for abilities in each occupation, w_R and w_S, the value of working in occupation i is denoted by $V_i(x_i, w_i)$ and it is equal to:

$$V_i(x_i, w_i) = \max_c \int_{y \in \mathbb{Y}} \frac{c^{1-\gamma}}{1-\gamma}\, dF_i(y) \tag{4}$$

$$\text{subject to } c \leq x_i\, e^y\, w_i$$

Among the two alternative careers, the worker picks the one with the highest value.

$$V(\boldsymbol{X}, w_R, w_S) = \max\{V_R(x_R, w_R), V_S(x_R, w_S)\} \tag{5}$$

Given that only two occupations are available, worker sorting in our environment is summarized by the share p_R of workers choosing the risky occupation.

Proposition 2.1 *The share of workers choosing occupation R,p_R, is given by*

[1] $\alpha > 2$ so that variance is finite.

$$p_R = \frac{T_R^{\frac{\alpha}{(1-\rho)}} |\Omega_R(w_R)|^{\frac{\alpha}{(1-\rho)(1-\gamma)}}}{\sum_{i \in \{R,S\}} T_i^{\frac{\alpha}{(1-\rho)}} |\Omega_i(w_i)|^{\frac{\alpha}{(1-\rho)(1-\gamma)}}} \tag{6}$$

where $\Omega_i = \int_{y \in \mathbb{Y}} \frac{(e^y w_i)^{1-\gamma}}{1-\gamma} \, dF_i(y)$.

Section A (online Appendix) offers a proof of this proposition. Using market wages, we determine the likelihood that the risky occupation's value exceeds the safe occupation's value for each ability level. Averaging these probabilities using the distribution of abilities in the safe occupation yields the expression.

Once we determine the probability of a worker selecting occupation R and thus the number of workers in this occupation, we define the abilities of these workers to calculate the total effective labor input.

Proposition 2.2 *The amount of efficiency units in occupation i is*

$$N_i = p_i \mathbb{E}(\tilde{x}_i) = p_i^{\frac{\alpha-(1-\rho)}{\alpha}} T_i \Gamma\left(1 - \frac{1}{\alpha}\right)$$

where $\mathbb{E}(\tilde{x}_i)$ *is the average ability of workers who choose occupation i (i.e. post-sorting).*

The result follows by first noting that $N_i = p_i \tilde{x}_i$, where $\tilde{x}_i$ is the average ability of a workers who choose occupation i. In section B of the online Appendix we offer a proof of this proposition. Note that $N_i = p_i^{\frac{-(1-\rho)}{\alpha}} p_i T_i \Gamma\left(1 - \frac{1}{\alpha}\right) = p_i^{\frac{-(1-\rho)}{\alpha}} \mathbb{E}(x_i)$ where $\mathbb{E}(x_i)$ is the average ex-ante ability (i.e. pre-sorting). Given that $\alpha > 2$ and $0 < \rho < 1$, it is easy to see that average skills of workers after sorting are higher than ex-ante average skills. This is the direct consequence of sorting given workers select based on their comparative advantage.

2.2 The Competitive Equilibrium Allocation

A competitive equilibrium is a pair of employment levels N_R and N_S, and a pair of wages w_R and w_S, and an associated level of output Y_{CE}. Employment depends on

workers' optimal choices, and wages match the marginal product of labor in each occupation due to perfect competition. Thus, using the expressions derived in Propositions 2.1 and 2.2, and the marginal products of each type of labor, we can derive closed-form expressions for N_R and N_S. Substituting into the production function we obtain the following result.

Proposition 2.3 *The competitive equilibrium level of output Y_{CE} is given by*

$$Y_{CE} = \left\{ \theta T_R^{\nu} \left[1 + \left(\frac{T_S}{T_R} \right)^{\frac{\alpha\nu((1-\rho)-\alpha)}{(\nu((1-\rho)-\alpha)+\alpha}((1-\rho)-\alpha)} \left(\frac{1-\theta}{\theta} \right)^{\frac{\alpha}{\nu((1-\rho)-\alpha)+\alpha}} \left(\frac{E_S}{E_R} \right)^{\frac{\alpha}{(\nu((1-\rho)-\alpha)+\alpha)(1-\gamma)}} \right]^{\frac{\nu((1-\rho)-\alpha)}{\alpha}} + \right.$$

$$\left. (1-\theta) T_S^{\nu} \left[1 + \left(\frac{T_R}{T_S} \right)^{\frac{-\alpha\nu((1-\rho)-\alpha)}{(\nu((1-\rho)-\alpha)+\alpha}((1-\rho)-\alpha)} \left(\frac{\theta}{1-\theta} \right)^{\frac{\alpha}{\nu((1-\rho)-\alpha)+\alpha}} \left(\frac{E_R}{E_S} \right)^{\frac{\alpha}{(\nu((1-\rho)-\alpha)+\alpha)(1-\gamma)}} \right]^{\frac{\nu((1-\rho)-\alpha)}{\alpha}} \right\}$$

$$\Gamma\left(1 - \frac{1}{\alpha}\right)$$

where $E_i = \mathbb{E}(e^{y_i(1-\gamma)}) = e^{(1-\gamma)(-\frac{\sigma_i^2\gamma}{2})}$

A detailed derivation of this result can be found in section C of the online Appendix.

3 The First-Best Allocation

We assess misallocation by comparing the competitive equilibrium to a frictionless economy with complete markets. We analyze resource allocation from a planner's perspective, who operates without constraints and aims to maximize output to maximize the welfare of a newborn with unknown abilities and shocks. The planner achieves maximum welfare by distributing resources evenly among all workers when the economy maximizes output (as there is no leisure nor savings).

Our planner allocates workers across the two occupations after observing each worker's ability. The planner does not observe the shocks that workers receive once they begin work in an occupation. We use Proposition 2.2 to solve the social planner's

problem, which reduces to finding the masses of workers in occupations R and S, p_R^{FB} and p_S^{FB} that maximize output.

$$\max_{p_R^{FB}, p_S^{FB}} \left[\theta T_R^{\nu} \left(p_R^{FB} \right)^{\nu \frac{\alpha-(1-\rho)}{\alpha}} + (1-\theta) T_S^{\nu} \left(p_S^{FB} \right)^{\nu \frac{\alpha-(1-\rho)}{\alpha}} \right]^{1/\nu} \Gamma\left(1 - \frac{1}{\alpha}\right) \tag{7}$$

subject to,

$$p_R^{FB} + p_S^{FB} = 1 \tag{8}$$

From the first order conditions we can solve for p_R^{FB} and p_S^{FB} in closed form. We then use Proposition 2.2 and the production function to obtain the efficient output, given by:

$$\begin{aligned} Y_{FB} = \Bigg[\theta T_R^{\nu} \left(\frac{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}}}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}} + 1} \right)^{\nu \frac{\alpha-(1-\rho)}{\alpha}} + \\ (1-\theta) T_R^{\nu} \left(\frac{1}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}} + 1} \right)^{\nu \frac{\alpha-(1-\rho)}{\alpha}} \Bigg]^{1/\nu} \Gamma\left(1 - \frac{1}{\alpha}\right) \end{aligned} \tag{9}$$

In Section D of the online Appendix we provide more details about the derivation.

3.1 Discussion

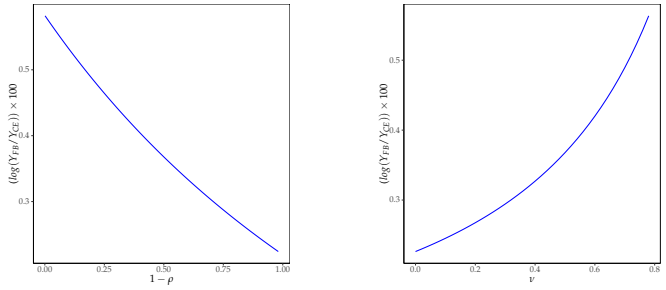
How does misallocation change with preferences or abilities? In Figure 1 we plot the log of the ratio of Y_{FB}/Y_{CE} (in percentage terms) for different values of the parameters of interest.

Recall that ρ governs the degree of comparative advantage. For instance, when ρ is near one, a worker excelling in one occupation likely excels in the other, leading to minimal worker selection based on abilities. Worker selection mitigates misallocation.

As the elasticity of substitution in production, denoted by ν, increases, so does misallocation (second figure). This is because higher substitutability between two

occupations leads to a smaller wage premium, due to wages being less responsive to worker allocation changes. Consequently, in a competitive equilibrium, the safer occupation becomes disproportionately large, resulting in lower overall output. Conversely, when elasticity decreases, the riskier occupation offers higher wages, attracting more workers and altering the allocation.

Figure 1



Misallocation and $(1-\rho)$. Misallocation and ν.

Notes: The two figures show how the degree of misallocation varies for different values of three parameters: (a) ρ, (b) ν. Misallocation is measured by the percentage deviation of the competitive equilibrium output (Y_{CE}) from the first best (Y_{FB}).

4 Conclusions

The absence of insurance markets against permanent earnings shocks influences workers' occupational choices and talent allocation, impacting aggregate productivity. In competitive equilibria, talent is misallocated as workers shy away from risky occupations, while a social planner would allocate more workers to these roles. The extent of misallocation increases with risk aversion and decreases with comparative advantage.

This paper presents a novel perspective on the relationship between labor market risks and overall human capital levels. We simplify labor market dynamics and individual career choices, acknowledging that many real-world barriers affect occupational choices and mobility. Our findings aim to spur further research that considers these factors.

Declaration of Interest

None

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Online Appendix (Not for Publication)

A Proof of Proposition 2.1

Proof To verify that expression, note that $p_R = Prob(V_R > V_S)$. We can rewrite $V_i(x_i, w_i)$ as,

$$V_i(x_i, w_i) = x_i^{1-\gamma} \int_{y \in \mathbb{Y}} \frac{(e^y w_i)^{1-\gamma}}{1-\gamma} \, dF_i(y) \tag{10}$$

Relabeling the integral as Ω_i, further rewrite $V_i(x_i, w_i)$ as $x_i^{1-\gamma}\Omega_i$. Note that $V_i(x_i, w_i) < 0$ for any $x_i, w_i > 0$. Since the occupational choice entails picking the maximum between $V_R(x_R, w_R)$ and $V_S(x_S, w_S)$, the choice is equivalent to choosing the minimum between $|V_R(x_R, w_R)|$ and $|V_S(x_S, w_S)|$. Therefore, $Pr(V_R > V_S) = Pr(|V_R| < |V_S|) = Pr(x_R^{1-\gamma}|\Omega_R| < x_S^{1-\gamma}|\Omega_S|) = Pr(x_R^{1-\gamma} < x_S^{1-\gamma}\frac{|\Omega_S|}{|\Omega_R|})$. Since $\gamma > 1$, [2]

$Pr(V_R > V_S) = Pr\Big(x_R(|\Omega_R|/|\Omega_S|)^{1/(1-\gamma)} > x_S\Big) = \int_0^\infty F_{x_R}(x, x(|\Omega_R|/|\Omega_S|)^{1/(1-\gamma)})dx.$
The derivative of the joint cumulative density function (2) with respect to x_R is,

$$\begin{aligned} F_{x_R}(x_R, x_S) = \exp\left\{-\Big[\sum_{i\in R,S}(T_i^{\alpha/(1-\rho)}x_i^{-\alpha/(1-\rho)})\Big]^{(1-\rho)}\right\} \\ \Big[\sum_{i\in R,S}(T_i^{\alpha/(1-\rho)}x_i^{-\alpha/(1-\rho)})\Big]^{-\rho}\alpha T_R^{\alpha/(1-\rho)}x_R^{-\alpha/(1-\rho)-1} \end{aligned} \tag{11}$$

Substituting for $x_R = x$ and $x_S = x\frac{|\Omega_R|}{|\Omega_S|}^{1/(1-\gamma)}$, defining κ_i to be $\frac{|\Omega_R|}{|\Omega_i|}^{1/(1-\gamma)}$ and integrating gives,[3]

$$\int F_{x_R}(x, x(|\Omega_R|/|\Omega_S|)^{1/(1-\gamma)}dx =$$

$$= \int \exp\left\{-\Big[\sum_{i\in R,S}\left(\frac{x\kappa_i}{T_i}\right)^{-\alpha/(1-\rho)}\Big]^{(1-\rho)}\right\}\Big[\sum_{i\in R,S}\left(\frac{x\kappa_i}{T_i}\right)^{-\alpha/(1-\rho)}\Big]^{-\rho}\alpha T_R^{\frac{\alpha}{(1-\rho)}}x^{-\frac{\alpha}{(1-\rho)}-1}dx =$$

[2] To understand the next equality, note that

$$F_{x_R}(x_R, x_S) = \frac{d}{dx_R}\int_0^{x_R}\int_0^{x_S} f(z, w)dzdw = \int_0^{x_S} f(z, x_R)dz.$$

We use standard notation $f(x_R, x_S)$ for the joint probability density function.

[3] The lower and upper integration limits are understood to be 0 and ∞.

$$= \int \exp\left\{-\left[\sum_{i\in R,S}\left(\frac{x\kappa_i}{T_i}\right)^{-\alpha/(1-\rho)}\right]^{(1-\rho)}\right\}\left[\sum_{i\in R,S}\left(\frac{\kappa_i}{T_i}\right)^{-\frac{\alpha}{(1-\rho)}}\right]^{-\rho}\alpha T_R^{\frac{\alpha}{(1-\rho)}}x^{\frac{-\alpha}{(1-\rho)}(-\rho)}x^{-\frac{\alpha}{(1-\rho)}-1}dx =$$

$$= \left[\sum_{i\in R,S}\left(\frac{\kappa_i}{T_i}\right)^{-\frac{\alpha}{(1-\rho)}}\right]^{-1} T_R^{\frac{\alpha}{(1-\rho)}}\int \exp\left\{-\left[\sum_{i\in R,S} T_i^{\frac{\alpha}{(1-\rho)}}\kappa_i^{-\frac{\alpha}{(1-\rho)}}x^{-\frac{\alpha}{(1-\rho)}}\right]^{(1-\rho)}\right\}$$

$$\left[\sum_{i\in R,S}\left(\frac{\kappa_i}{T_i}\right)^{-\frac{\alpha}{(1-\rho)}}\right]^{(1-\rho)}\alpha x^{-\alpha-1}dx =$$

$$= \left[\sum_{i\in R,S}\left(\frac{\kappa_i}{T_i}\right)^{-\frac{\alpha}{(1-\rho)}}\right]^{-1} T_R^{\frac{\alpha}{(1-\rho)}}\int f(x)dx = T_R^{\frac{\alpha}{(1-\rho)}}\left[\sum_{i\in R,S}\left(\frac{\kappa_i}{T_i}\right)^{-\frac{\alpha}{(1-\rho)}}\right]^{-1} \tag{12}$$

Since κ_i equals $\frac{|\Omega_R|}{|\Omega_i|}^{1/(1-\gamma)}$ for $i \in \{R, S\}$, substitution yields,

$$p_R = \frac{T_R^{\frac{\alpha}{(1-\rho)}}|\Omega_R(w_R)|^{\frac{\alpha}{(1-\rho)(1-\gamma)}}}{\sum_{i\in\{R,S\}} T_i^{\frac{\alpha}{(1-\rho)}}|\Omega_i(w_i)|^{\frac{\alpha}{(1-\rho)(1-\gamma)}}} \tag{13}$$

B Proof of Proposition 2.2

Proof We denote by $\tilde{x}_i$ the average ability of a workers who choose occupation i. Given that shocks that workers experience after they have chosen an occupation are of mean equal to one, the amount of efficiency units in occupation $i \in \{R, S\}$ is given by $N_i = p_i\tilde{x}_i$. The distributional assumption on the joint distribution of $\boldsymbol{X} = (x_R, x_S)$ implies that the post-sorting distribution of abilities is also Fréchet.

To derive this result we begin by defining the extreme value $V^* = \min_i\left\{x_i^{1-\gamma}|\Omega_i|\right\}$. As a result for a given $b > 0$, $Pr(V^* > b) = Pr(x_i^{1-\gamma}|\Omega_i| > b) = Pr(x_i^{1-\gamma} > b/|\Omega_i|)$ for all i, which in turn equals,

$$Pr\left(x_i < \left(\frac{b}{|\Omega_i|}\right)^{1/(1-\gamma)}\right) \text{ for all } i.$$

Using the joint cdf, that probability is given by,

$$F\left(\frac{b}{|\Omega_R|}, \frac{b}{|\Omega_S|}\right) = \exp\left\{-\left[\sum_{i\in R,S} T_i^{\frac{\alpha}{(1-\rho)}}\left(\frac{b}{|\Omega_i|}\right)^{\frac{-\alpha}{(1-\rho)(1-\gamma)}}\right]^{(1-\rho)}\right\} =$$

$$= \exp\left\{-\left[\sum_{i\in R,S}\left(T_i^{\frac{\alpha}{(1-\rho)}}|\Omega_i|^{\frac{\alpha}{(1-\rho)(1-\gamma)}}b^{\frac{-\alpha}{(1-\rho)(1-\gamma)}}\right)\right]^{(1-\rho)}\right\} =$$

$$= \exp\left\{-\left[\hat{T}^{(1-\rho)}\left(b^{\frac{-\alpha}{(1-\rho)(1-\gamma)}}\right)^{(1-\rho)}\right]\right\}. \tag{14}$$

where $\hat{T} = \sum_{i \in R,S} T_i^{\frac{\alpha}{(1-\rho)}} |\Omega_i|^{\frac{\alpha}{(1-\rho)(1-\gamma)}}$. Since $Pr(V^* > b) = 1 - Pr(V^* < b)$, the cdf of V^* is given by,

$$Pr(V^* < b) = 1 - \exp\left\{-\left[\hat{T}^{(1-\rho)} b^{-\alpha/(1-\gamma)}\right]\right\}. \tag{15}$$

Note that this is the distribution for the extreme value $V^* = x^{*1-\gamma}|\Omega^*| = \min_i x_i^{1-\gamma}|\Omega_i|$. We are interested in the cdf of x^*, the distribution of abilities post-sorting. To obtain that distribution, note that $Pr(V^* > b) = Pr\left(x^* < \left(\frac{b}{|\Omega^*|}\right)^{1/(1-\gamma)}\right) = Pr(x^* < b^*)$ Using the first term in (14), that probability is given by,

$$Pr(x^* < b^*) = \exp\left\{-\left[\sum_{i \in R,S} T_i^{\frac{\alpha}{(1-\rho)}} \left(\frac{b}{|\Omega_i|}\right)^{\frac{-\alpha}{(1-\rho)(1-\gamma)}}\right]^{(1-\rho)}\right\} =$$

$$= \exp\left\{-\left[T^{*\frac{-(1-\rho)}{\alpha}} b^*\right]^{-\alpha}\right\} \tag{16}$$

where $T_i^* = \sum_{i \in R,S} T_i^{\frac{\alpha}{(1-\rho)}} \left(\frac{|\Omega_i^*|}{|\Omega_i|}\right)^{\frac{-\alpha}{\rho(1-\gamma)}}$.

Equation (16) shows that the distribution of x^*, the ability of workers who have chosen an occupation, is Fréchet. Its shape parameter is equal to α and its scale parameter is $T^{*\frac{(1-\rho)}{\alpha}}$. The mean of this distribution is $T^{*\frac{(1-\rho)}{\alpha}}\Gamma(1-\frac{1}{\alpha})$.

By letting $|\Omega_i^*| = |\Omega_i|$, we have that

$$T_i^* = T_i^{\frac{\alpha}{(1-\rho)}}/p_i$$

. Thus, the mean of that distribution can be written as $T_i p_i^{\frac{-(1-\rho)}{\alpha}}\Gamma(1-\frac{1}{\alpha})$. For occupation R, it is given by,

$$\tilde{x}_R = E(x_R) = T_R p_R^{\frac{-(1-\rho)}{\alpha}}\Gamma(1-1/\alpha), \tag{17}$$

And for occupation S by,

$$\tilde{x}_S = E(x_S) = T_S p_S^{\frac{-(1-\rho)}{\alpha}}\Gamma(1-1/\alpha), \tag{18}$$

Once we have $E(\tilde{x}_1)$ and $E(\tilde{x}_2)$ the result follows:

$$N_i = p_i\tilde{x}_i = T_i p_i^{\frac{\alpha-(1-\rho)}{\alpha}}\Gamma(1-1/\alpha), \tag{19}$$

C Proof of Proposition 2.3

To begin, note that from by combining 2.1 and 2.2, N_i, $i \in \{R, S\}$ equals

Proof

$$N_i = T_i p_i^{\frac{\alpha-(1-\rho)}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) = T_i \left[\frac{T_i^{\frac{\alpha}{(1-\rho)}} \Omega_i^{\frac{\alpha}{(1-\rho)(1-\gamma)}}}{T_R^{\frac{\alpha}{(1-\rho)}} \Omega_R^{\frac{\alpha}{(1-\rho)(1-\gamma)}} + T_S^{\frac{\alpha}{(1-\rho)}} \Omega_S^{\frac{\alpha}{(1-\rho)(1-\gamma)}}}\right]^{\frac{\alpha-(1-\rho)}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) =$$

$$T_i \left[\sum_{j\in\{R,S\}} \left(\frac{T_j}{T_i}\right)^{\frac{\alpha}{(1-\rho)}} \left(\frac{\Omega_j}{\Omega_i}\right)^{\frac{\alpha}{(1-\rho)(1-\gamma)}}\right]^{\frac{(1-\rho)-\alpha}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) \tag{20}$$

Also note that the ratio of the two labor inputs in efficiency units is,

$$\frac{N_R}{N_S} = \frac{T_R}{T_S}\left(\frac{T_R^{\frac{\alpha}{(1-\rho)}} \Omega_R^{\frac{\alpha}{(1-\rho)(1-\gamma)}}}{T_S^{\frac{\alpha}{(1-\rho)}} \Omega_S^{\frac{\alpha}{(1-\rho)(1-\gamma)}}}\right)^{\frac{\alpha-(1-\rho)}{\alpha}} = \left(\frac{T_R}{T_S}\right)^{\frac{\alpha}{(1-\rho)}} \left(\frac{\Omega_R}{\Omega_S}\right)^{\frac{\alpha-(1-\rho)}{(1-\rho)(1-\gamma)}}$$

$$= \left(\frac{T_R}{T_S}\right)^{\frac{\alpha}{(1-\rho)}} \left(\frac{w_R^{1-\gamma} E_R}{w_S^{1-\gamma} E_S}\right)^{\frac{\alpha-(1-\rho)}{(1-\rho)(1-\gamma)}} \tag{21}$$

where $E_i = \mathbb{E}(e^{y_i(1-\gamma)})$. In equilibrium, wages are equal to the marginal products of the two types of labor. Given our aggregate technology,

$Y = \left[\theta N_R^\nu + (1-\theta) N_S^\nu\right]^{1/\nu}$

we have that

$w_R = \left[\theta N_R^\nu + (1-\theta) N_S^\nu\right]^{1/\nu-1} \theta N_R^{\nu-1}$ and $w_S = \left[\theta N_R^\nu + (1-\theta) N_S^\nu\right]^{1/\nu-1} (1-\theta) N_S^{\nu-1}$.

Thus,

$$\frac{w_R}{w_S} = \left(\frac{\theta}{1-\theta}\right)\left(\frac{N_R}{N_S}\right)^{\nu-1} \tag{22}$$

Substituting (22) into (21), we get

$$\frac{N_R}{N_S} = \left(\frac{T_R}{T_S}\right)^{\frac{\alpha}{(1-\rho)}} \left(\frac{\theta}{1-\theta}\right)^{\frac{\alpha-(1-\rho)}{(1-\rho)}} \left(\frac{N_R}{N_S}\right)^{-(\nu-1)\frac{(1-\rho)-\alpha}{(1-\rho)}} \left(\frac{E_R}{E_S}\right)^{\frac{\alpha-(1-\rho)}{(1-\rho)(1-\gamma)}} \tag{23}$$

Simplifying

$$\frac{N_R}{N_S} = \left(\frac{T_R}{T_S}\right)^{\frac{\alpha}{\nu((1-\rho)-\alpha)+\alpha}} \left(\frac{\theta}{1-\theta}\right)^{\frac{\alpha-(1-\rho)}{\nu((1-\rho)-\alpha)+\alpha}} \left(\frac{E_R}{E_S}\right)^{\frac{\alpha-(1-\rho)}{(\nu((1-\rho)-\alpha)+\alpha)(1-\gamma)}} \tag{24}$$

Note from (20) that N_R is,

$$N_R = T_R \left[1 + \left(\frac{T_S}{T_R}\right)^{\frac{\alpha}{(1-\rho)}} \left(\frac{\Omega_S}{\Omega_R}\right)^{\frac{\alpha}{(1-\rho)(1-\gamma)}}\right]^{\frac{(1-\rho)-\alpha}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) \tag{25}$$

$$= T_R \left[1 + \left(\frac{T_S}{T_R}\left(\frac{\Omega_R}{\Omega_S}\right)^{\frac{1}{(\gamma-1)}}\right)^{\frac{\alpha}{(1-\rho)}}\right]^{\frac{(1-\rho)-\alpha}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) \tag{26}$$

and from (21)

$\frac{N_R}{N_S} = \frac{T_R}{T_S}\left(\frac{T_S}{T_R}\left(\frac{\Omega_R}{\Omega_S}\right)^{\frac{1}{(\gamma-1)}}\right)^{\frac{(1-\rho)-\alpha}{(1-\rho)}}$ so that, $\frac{T_S}{T_R}\left(\frac{\Omega_R}{\Omega_S}\right)^{\frac{1}{(\gamma-1)}} = \left(\frac{T_S}{T_R}\frac{N_R}{N_S}\right)^{\frac{(1-\rho)}{(1-\rho)-\alpha}}$.

Substituting back into (26),

$$N_R = T_R \left[1 + \left(\frac{T_S N_R}{T_R N_S}\right)^{\frac{\alpha}{(1-\rho)-\alpha}}\right]^{\frac{(1-\rho)-\alpha}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) =$$

$$\left[1 + \left(\frac{T_S}{T_R}\right)^{\frac{\alpha}{(1-\rho)-\alpha}}\left(\frac{N_R}{N_S}\right)^{\frac{\alpha}{(1-\rho)-\alpha}}\right]^{\frac{(1-\rho)-\alpha}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) \tag{27}$$

Substituting for the value of the ratio of labor inputs given by (24)

$$N_R = T_R \left[1 + \left(\frac{T_S}{T_R}\left(\frac{T_S}{T_R}\right)^{\frac{-\alpha}{\nu((1-\rho)-\alpha)+\alpha}}\left(\frac{\theta}{1-\theta}\right)^{\frac{\alpha-(1-\rho)}{\nu((1-\rho)-\alpha)+\alpha}}\left(\frac{E_R}{E_S}\right)^{\frac{\alpha-(1-\rho)}{(\nu((1-\rho)-\alpha)+\alpha)(1-\gamma)}}\right)^{\frac{\alpha}{(1-\rho)-\alpha}}\right]^{\frac{(1-\rho)-\alpha}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) \tag{28}$$

Further simplification gives,

$$N_R = T_R \left[1 + \left(\frac{T_S}{T_R}\right)^{\frac{\alpha\nu((1-\rho)-\alpha)}{(\nu((1-\rho)-\alpha)+\alpha)((1-\rho)-\alpha)}}\left(\frac{1-\theta}{\theta}\right)^{\frac{\alpha}{\nu((1-\rho)-\alpha)+\alpha}}\left(\frac{E_S}{E_R}\right)^{\frac{\alpha}{(\nu((1-\rho)-\alpha)+\alpha)(1-\gamma)}}\right]^{\frac{(1-\rho)-\alpha}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) \tag{29}$$

Similarly for N_S we have,

$$N_S = T_S \left[1 + \left(\frac{T_S}{T_R}\right)^{\frac{-\alpha\nu((1-\rho)-\alpha)}{(\nu((1-\rho)-\alpha)+\alpha)((1-\rho)-\alpha)}}\left(\frac{1-\theta}{\theta}\right)^{\frac{-\alpha}{\nu((1-\rho)-\alpha)+\alpha}}\left(\frac{E_S}{E_R}\right)^{\frac{-\alpha}{(\nu((1-\rho)-\alpha)+\alpha)(1-\gamma)}}\right]^{\frac{(1-\rho)-\alpha}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) \tag{30}$$

By substituting the expressions for N_R and N_S into the production function we obtain the competitive equilibrium level of output Y_{CE}.

D The First-Best Allocation

We equalize the first order conditions for this problem render (note that the term containing the Γ function cancels out because it is a constant):

$$\theta T_R^{\nu} \left(p_R^{FB}\right)^{\nu\frac{\alpha-(1-\rho)}{\alpha}-1} = T_S^{\nu}(1-\theta)\left(p_S^{FB}\right)^{\nu\frac{\alpha-(1-\rho)}{\alpha}-1} \tag{31}$$

Since the two masses have to add up to one, we get that

$$p_R^{FB} = \frac{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}}}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}}+1} \text{ and } p_S^{FB} = \frac{1}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}}+1}.$$

Plugging back into the definition of efficiency units we get the allocation of efficiency units chosen by the social planner:

$$N_R^{FB} = T_R \left[\frac{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}}}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}}+1}\right]^{\frac{\alpha-(1-\rho)}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) \tag{32}$$

$$N_S^{FB} = T_S \left[\frac{1}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}}+1}\right]^{\frac{\alpha-(1-\rho)}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) \tag{33}$$

Given the labor inputs chosen by the planner, the efficient level of output is

$$\begin{aligned} Y_{FB} = \Bigg[\theta T_R^{\nu} \left(\frac{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}}}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}}+1}\right)^{\nu\frac{\alpha-(1-\rho)}{\alpha}} + \\ (1-\theta)T_R^{\nu}\left(\frac{1}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}}+1}\right)^{\nu\frac{\alpha-(1-\rho)}{\alpha}}\Bigg]^{1/\nu} \Gamma\left(1-\frac{1}{\alpha}\right) \end{aligned} \tag{34}$$