Wealth Inequality and Employment Fluctuations*

Enchuan Shao[†] Pedro Silos[‡]
University of Saskatchewan Temple University

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Abstract

This paper is concerned with the business cycle dynamics in search and matching models of the labor market when agents are ex-post heterogeneous. We focus on heterogeneity caused by different labor market histories and the resulting wealth inequality they generate. We show that this inequality implies wage rigidity relative to a complete insurance economy. The fraction of wealth poor agents prevents real wages from falling too much in recessions, since small decreases in income imply large losses in utility. Analogously, wages rise less during expansions than in models with homogeneous workers as small increases are enough for poor workers to accept job offers. This mechanism reduces the volatility of wages but generates more volatile employment levels.

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[†]Department of Economics, University of Saskatchewan, Arts 811, 9 Campus Dr, Saskatoon, SK S7N5A5, Canada, Email: enchuan.shao@usask.ca

[‡]Department of Economics, Temple University, 1301 Cecil B. Moore Ave, Ritter Annex 879, Philadelphia, PA 19122, Email: pedro.silos@temple.edu

1 Introduction

The pool of unemployed individuals at any point in time and across countries displays considerable heterogeneity. Workers searching for jobs are different in terms of skill, age, wealth or health, and these differences affect both their search behavior and their bargaining position when, after meeting with a prospective employer, they negotiate the terms of their employment contract.[1] This paper focuses on one dimension along which the working and the unemployed differ - their level of wealth. Much of the existing literature on the macroeconomics of labor markets, makes wealth heterogeneity irrelevant by assuming either complete financial markets or preferences that make individuals neutral to income fluctuations. We construct an environment which features risk-averse agents who are subject to unemployment shocks; they can either have a job from which they can be displaced or find a job in case they are looking for one. Transitions in and out of unemployment generate income fluctuations against which agents can only self-insure by adjusting their stock of physical capital. Different unemployment histories generate different income histories, resulting in different wealth levels across agents. We find that accounting for *individual* wealth heterogeneity matters for *aggregate* fluctuations in employment, output, and wages.

More specifically, we find that the shape of the distribution of wealth, and in particular the fraction of agents close to the borrowing constraint, matters for aggregate fluctuations and most importantly for the degree of wage rigidity. Higher wage rigidity implies larger fluctuations in employment and vacancies: increases in the productivity of workers will lead to more hiring the less wages adjust to productivity increases. The reason why a large fraction of wealth-poor agents would lead to relatively more rigid wages is quite intuitive. When the negotiation of wages takes place, a large fraction of agents close to the borrowing constraint prevents wages from falling too much during a recession. The reason is that small decreases in the real wage imply large losses in utility. Analogously, during an expansion a mild increase in wages is enough for very poor agents to accept a job offer, as their utility increases substantially. Firms react by posting more vacancies during booms and fewer during downturns than they would otherwise.

[1] Empirical analysis provided by Chetty (2008) has shown that, for instance, the effect of unemployment insurance on unemployment durations is larger for borrowing-constrained than for unconstrained individuals.

The model economy we present is a version of the stochastic growth model with labor search and matching frictions. Firms post job vacancies and workers search when they are unemployed hoping to get matched to a job offer. Employed workers are at risk of losing their job and becoming unemployed. However, we assume that there is no insurance mechanism that can perfectly eliminate the employment risk: agents have to self-insure using their holdings of physical capital only. Without additional frictions, our results show that, quantitatively, the ability of agents to smooth consumption effectively, precludes a large mass of them from being borrowing constrained. Fluctuations in the labor market look similar to those that obtain in a model with homogeneous agents. This feature of the wealth distribution in our model is consistent with Krusell and Smith (1998) work, where the lack of perfect insurance in a version of the stochastic growth model generates too few poor agents and many rich individuals. However, it is inconsistent with the actual wealth distribution in many developed countries, in particular that of the United States. Empirical studies have shown that the fraction of borrowing constrained households could be as high as 25% to 30% of all households. Given that the power of the mechanism outlined here is directly related to the mass of agents that are close to the borrowing constraint, we explore features that prevent agents from smoothing out shocks effectively and which result in a wealth distribution which is similar to its empirical counterpart. Specifically, we evaluate the effects of introducing (separately) the following features in the model: an irreversibility constraint on investment, heterogeneous discount factors, and different productivity levels across workers. All these versions imply very different dynamics of aggregate variables. In some cases, the improvement is quite significant. For instance, assuming a labor income distribution by augmenting the wage rate with a random productivity shock almost triples the volatility of the vacancy-unemployment ratio in comparison to the full insurance model.

There is by now a large literature on search and matching in the labor markets, having become the standard way of thinking about labor markets in models of aggregate fluctuations.That literature began with Andolfatto (1996) and Merz (1995) who assume that all workers belong to a household in which some agents work and others search. However, they all insure each other against being fired or not finding a job. Acemoglu and Shimer (1999) focus on the optimal unemployment insurance contract in a search environment with capital accumulation and where agents are risk averse. However, they do not introduce aggregate shocks. In a line of research more related to our paper, although developed independently, Rudanko (2009) and Rudanko (2011) build an economy in which agents face idiosyncratic and aggregate shocks. She introduces search and matching frictions in the labor market, and long term contracts in wages where the firm provides insurance to the worker against drops in productivity. She also assesses how changes in risk aversion or in the value of being unemployed affects the quantitative implications of heterogeneity for explaining the labor market business cycle facts. A key difference between hers and our paper is that there is no capital accumulation (or any form of savings) in her model. The worker consumes the wage and the unemployed consumes the unemployment benefit. Our model complements hers by introducing heterogeneity in a stochastic growth model with labor market search and production, therefore making our results more comparable to the real business cycle literature. Other close competitors to our paper are Costain and Reiter (2005), Krusell *et al.* (2010), and Nakajima (2012). They all introduce market incompleteness and self-insurance into the Mortensen-Pissarides framework, and assess their effects on aggregate fluctuations. Results are similar but there are interesting differences in modeling. For instance, Krusell *et al.* (2010) assume individual bargaining, whereas Costain and Reiter (2005) assume a form of "sectoral" bargaining, and we assume collective (aggregate) bargaining. Costain and Reiter's economy does not use capital and interest rates are fixed. Moreover, their focus is different, emphasizing the role of counter-cyclical fiscal policy. We, on the other hand, stress the implications of the shape of the wealth distribution for business cycle dynamics. In terms of financial markets, Krusell *et al.* (2010) distinguish between *ex-ante* return properties of capital holdings and firm shares. Alternatively, we model entrepreneurs as owners of firms and households making the capital investment decisions, therefore there are no firm shares available to the household to speak of. Nakajima (2012) highlights the importance of labor-leisure choice while in our paper, the labor supply is inelastic and we emphasize the importance of wealth inequality among workers. Nevertheless, results seem to be robust to these modeling differences.[2]

The work by Shimer (2005) has been followed by numerous studies that hope to improve the ability of the Mortensen-Pissarides framework to be consistent with the

[2]A similar paper from the modeling perspective but with a different focus is Bils *et al.* (2012). In an economy without capital, they analyze the exiting and search behavior of workers with different levels of human capital.

labor market business cycle facts. For example, Hagedorn and Manovskii (2008) have shown that the model presented in Shimer (2005) matches the volatility of the market tightness if it is calibrated in a particular way. Specifically, they show how making the outside option for a worker very valuable can improve the model's implications along several dimensions. However, other authors have pointed out additional problems with the Hagedorn and Manovskii's calibration (see, for example, the survey by Hornstein *et al.* (2005)). Hall (2005) shows how wage stickiness affects the cyclical behavior of unemployment in a Mortensen-Pissarides framework. In his study, wage stickiness is an equilibrium outcome in the sense that it does not affect the efficiency of the bargaining process between workers and firms.

2 The Model

2.1 Economic Environment

The model is a version of the one-sector stochastic growth model with labor market search frictions and where opportunities for perfect insurance are absent. There is a continuum of agents distributed uniformly on the unit interval. They are all endowed with one unit of time and maximize expected lifetime utility of consumption $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$, where U satisfies the usual conditions and β is a factor of time preference. Each agent faces different opportunities for exchanging labor services. In particular, individuals either have a job opportunity or they do not, and job opportunities arrive at random as is typical in the standard labor market search model. The absence of a full set of contingent claims implies that an agent's employment status determines his income. To smooth consumption across states and time, agents can only use physical capital k and they are all endowed with k_0 of it to start with. The initial employment status $i \in \{u, e\}$ is also given, where u denotes unemployed and e being employed.

There is a continuum of risk neutral entrepreneurs who maximize $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \phi_t$, where ϕ is the sum of current period cash flows from firms that they own.[3] Firms use capital K and labor N to produce output Y subject to a constant returns-to-scale production technology $Y = zF(K, N)$. The aggregate productivity z of firms evolves

[3] In principle, ϕ_t could be negative. However, this was not the case in any of our simulations.

according to a stochastic process known by agents.

In order to produce output, each job requires a worker. Let N_t denote the number of jobs that are matched with a worker at the beginning of period t; hence, N_t is the measure of current period employed workers and $1 - N_t$ is the measure of unemployed workers currently available for work. Let V_t denote the total number of new jobs made available by firms during period t. Following Pissarides (2001), the rate at which new job matches are formed is governed by an aggregate matching technology, $M(V_t, 1 - N_t)$, so that the employment evolves according to:

$$N_{t+1} = (1 - s) N_t + M_t,$$

where $s \in (0, 1)$ is the exogenous separation rate of job-worker pairs.[4] The probability for a worker to find a job offer is $\pi_t = M(V_t, 1 - N_t) / (1 - N_t)$ and the probability for a firm to match a worker with a vacancy is $p_t = M(V_t, 1 - N_t) / V_t$.

The the timing of events can be summarized as follows.

1. At the beginning of period t, the aggregate productivity shock z_t is revealed and publicly observed.
2. Goods and capital markets open.
   (a) A representative firm rents capital from both types of agents (workers and searchers), uses N_t units of labor to produce output, and posts new job vacancies V_t.
   (b) The worker provides labor services to the firm and, in return, receives wage payments which are determined by a bargaining rule. Besides labor income, the worker also receives interest payments on capital and makes consumption and investment decisions.
   (c) The unemployed individual receives no wage income and finances consumption and investment decisions with unemployment benefits and capital interest payments.
3. Goods and capital markets close and the labor market opens:

[4] The separation rate s does not depend on the stage of the business cycle. Shimer (2012) using CPS data finds that separation rates are approximately acyclical.

(a) The unemployed individuals and the firm search in the labor market. If they are successfully matched a new job is created which will be filled the following period.

(b) The employed agents might be separated from their current match with probability s. They must wait until the following period to search for work.

(c) The workers who remain employed and those who are successfully matched with the firm constitute a class of employed workers the following period.

4. The labor market closes.

2.2 Optimization

The agents' employment status is determined by whether they successfully matched with a firm the previous period (in case they were unemployed) and whether they were exogenously separated (in case they were employed). This random matching and separation process induces different employment histories among agents and consequently leads to heterogeneous asset holdings. Let $Q_t(k,i)$ denote the joint distribution of individual capital holdings and employment status at period t. This cross-sectional distribution evolves according to the law of motion:

$$Q_{t+1} = H(Q_t, z_t).$$

The set of state variables in the agents' problem consists of k_t and $\chi_t \equiv \{z_t, Q_t\}$. An agent chooses a level of consumption c^i, and saving k^i contingent upon the agent's employment status i. The measure of unemployed and employed workers can be obtained by integrating Q over the appropriate type,

$$\begin{aligned} N_t &= \int_{i=e} Q_t(k, di), \\ 1 - N_t &= \int_{i=u} Q_t(k, di). \end{aligned}$$

We now switch notation slightly and we will denote variables with no subscript to be current period variables and variables with a prime to be next period's variables. Denoting by J^e the value function for an employed worker and J^u the value function for an unemployed worker, the Bellman equation for an agent who works during the current period is:

$$J^e(k,\chi) = \max_{\{c^e,k^{e\prime}\}} U(c^e) + \beta\left[(1-s)\,\mathbb{E}J^e(k^{e\prime},\chi') + s\mathbb{E}J^u(k^{e\prime},\chi')\right] \tag{1}$$

$$s.t. \qquad c^e + k^{e\prime} = w + Rk + (1-\delta)\,k, \tag{2}$$

$$k^{e\prime} \geq \underline{k}, Q' = H(\chi)\,. \tag{3}$$

The value function of the worker is determined by the wage she obtains the current period plus the capital income obtained by renting capital. The worker takes into account that she might be unemployed in the following period with probability s and be employed with probability $1-s$. The constraints in this optimization problem are the budget constraint for the employed worker, a borrowing constraint with borrowing limit $\underline{k} \geq 0$, and a law of motion for the aggregate distribution of asset holdings and employment status. The wage rate w is determined by a bargaining rule to be discussed later and the interest rate R is determined in a competitive financial market.

Analogously the Bellman equation for an agent who searches the current period is:

$$J^u(k,\chi) = \max_{\{c^u,k^{u\prime}\}} U(c^u) + \beta\left[(1-\pi)\,\mathbb{E}J^u(k^{u\prime},\chi) + \pi\mathbb{E}J^e(k^{u\prime},\chi')\right] \tag{4}$$

$$s.t. \qquad c^u + k^{u\prime} = b + Rk + (1-\delta)\,k, \tag{5}$$

$$k^{u\prime} \geq \underline{k}, Q' = H(\chi)\,. \tag{6}$$

An unemployed agent receives unemployment income b which, along capital income, finances her consumption and investment expenditures. Following with Krusell *et al.* (2010), we introduce b as home production, and interpret b as unemployment insurance as in Hansen and Imrohoroglu (1992). In computing her expected value function she takes into account a probability π of being matched with a firm this period and working the following period and a probability $1-\pi$ of remaining unemployed.

A firm which is owned by an entrepreneur maximizes the present discounted value of the stream of future profits. A firm interested in filling an available job must undertake recruiting and screening activities, which are necessary for finding a suitable employee. Denoting by ω the unitary cost of recruiting, the firm chooses a contingency plan of vacancies and capital $\{V_t, K_t\}_{t=0}^{\infty}$ that maximizes the expected discounted sum of cash flows. The Bellman equation for this maximization problem is:

$$W(\chi) = \max_{\{V,K\}} \left\{ zK^{\alpha}N^{1-\alpha} - RK - wN - \omega V + \beta \mathbb{E} W(\chi') \right\} \tag{7}$$

$$s.t. \qquad N' = (1-s)N + pV, \tag{8}$$

$$Q' = H(\chi). \tag{9}$$

In the firm's Bellman equation we explicitly differentiate between the capital demanded by the firm, K, and the capital supplied by the individuals, implicit in the state vector χ. After equating these two in equilibrium, the optimal choices for the firm are given by the following optimality conditions:

$$R = \alpha z K^{\alpha-1} N^{1-\alpha}, \tag{10}$$

$$\frac{\omega}{p} = \beta \mathbb{E}\left[(1-\alpha) z' K'^{\alpha} N'^{-\alpha} - w' + \frac{\omega(1-s)}{p'} \right]. \tag{11}$$

2.3 Wage Bargaining

In principle, bargaining should occur between each worker and the firm, as heterogeneity in asset holdings implies heterogeneity in outside options. This individual negotiation would result in a distribution of individual-specific wages. The purpose of this paper is not to understand wage dispersion or the dynamics of the income distribution, and therefore we assume that workers can form a labor union.[5] The firm negotiates with the union rather than with individual workers.[6] The objective of the labor union is to maximize the aggregate surplus of all agents, which is given by,

$$\int \left[J^e(k,\chi) - J^u(k,\chi) \right] Q(dk, di) =$$

$$N \int_e \left[J^e(k,\chi) - J^u(k,\chi) \right] Q(dk, e) + (1-N) \int_u \left[J^e(k,\chi) - J^u(k,\chi) \right] Q(dk, u) \ .$$

[5]This assumption is similar to the one made in Costain and Reiter (2005). In their economy, firms negotiate by their vacancy status. Firms with vacant jobs bargain differently than firms with filled jobs. Here we assume a representative firm so that bargaining is done at the aggregate level, both for the firm and for the households.

[6]Several papers have estimated the effect of individual asset holdings on job market outcomes. Rendon (2006), using a structural search model, finds that higher wealth allows richer job seekers to be more selective, resulting in higher reservation wages. Similar results are found in Bloemen and Stancanelli (2001), who find higher wealth correlated with higher reservation wages, but small effects on job finding probabilities. For tractability, we abstract from studying these effects.

The symbol $\int_i, i \in \{e, u\}$ means integrating over assets held by either employed or unemployed agents. The previous expression deserves further explanation. For any agent, whether employed or not, the outside option – threat point – from being employed is the utility of becoming unemployed. It is important to note that $J^e(k,\chi) - J^u(k,\chi)$ is not the utility of the currently employed minus the utility of the currently unemployed. On the right hand side, since the integral is over the distribution of the currently employed (or unemployed), the surplus for each group, is the difference in the value function evaluated at the same combination of assets and aggregate states. The marginal value of a match for the firm is $\partial W(\chi)/\partial N$. The wage solves the following Nash bargaining problem:

$$\max_w \left(\frac{\partial W(\chi)}{\partial N}\right)^{\xi} \left(\int \left[J^e(k,\chi) - J^u(k,\chi)\right] Q(dk, di)\right)^{1-\xi},$$

where ξ is the firm's bargaining power. The Nash bargaining solution can be summarized as

$$\xi \left(\int \left[J^e(k,\chi) - J^u(k,\chi)\right] Q(dk, di) \right) = (1-\xi)\,\tilde{\Lambda} \frac{\partial W(\chi)}{\partial N}, \tag{12}$$

where $\tilde{\Lambda} = N \int_e U'(c^e(k,\chi))\, Q(dk, e) + (1 - N_t) \int_u U'(c^u(k,\chi))\, Q(dk, u)$ is the marginal payoff of being employed.

The marginal value of employment for the firm can be obtained from (7) and (8),

$$\frac{\partial W(\chi)}{\partial N} = (1-\alpha)\, z K^{\alpha} N^{-\alpha} - w + \frac{\omega(1-s)}{p}. \tag{13}$$

Substituting (13) into (12), we have the wage equation

$$w = (1-\alpha)\, z K^{\alpha} N^{-\alpha} + \frac{\omega(1-s)}{p} - \frac{\xi}{1-\xi} \frac{1}{\tilde{\Lambda}} \left(\int \left[J^e(k,\chi) - J^u(k,\chi)\right] Q(dk, di) \right). \tag{14}$$

Using (14) and (11), we can solve for the optimal job posting,

$$\frac{\omega}{p} = \beta \mathbb{E} \left[\frac{\xi}{1-\xi} \frac{1}{\tilde{\Lambda}'} \left(\int \left[J^e(k',\chi') - J^u(k',\chi')\right] Q'(dk', di') \right) \right]. \tag{15}$$

2.4 Equilibrium

Definition 1. A recursive competitive equilibrium is a pair of price functions R and w, the individual's value functions $J^u(k,\chi)$ and $J^e(k,\chi)$, decision rules $k^{i\prime}(k,\chi)$, $c^i(k,\chi)$ for $i \in \{u,e\}$, and vacancies posted V, and a law of motion H for Q such that

1. Given prices, the number of job vacancies V which determines the matching probability, and the law of motion H, the value functions $J^u(k,\chi)$ and $J^e(k,\chi)$ solves the agents' optimization problem; and the optimal decision rules are $k^{i\prime}(k,\chi)$, $c^i(k,\chi)$.

2. Given the decision rules, the optimal job posting rule V is determined by maximizing the firm's discounted present value of profits, i.e. V satisfies (15);

3. The interest rate R satisfies (10) and the wage rate is the solution to the Nash bargaining problem (14);

4. The decision rules and the Markov processes for z and s imply that today's distribution Q is mapped into tomorrow's Q' by H;

5. Goods market must clear:

$$\int cdQ + K' - (1-\delta)K + \omega V + \phi = zK^{\alpha}N^{1-\alpha} + b. \tag{16}$$

As is typical in models with idiosyncratic and aggregate risk, one needs to avoid having the entire distribution Q as a state variable in order to obtain quantitative results. As other examples in the literature do, we follow Krusell and Smith (1998) and others in summarizing the distribution Q by a vector of its moments m and replacing H by some polynomial that determines m' as a function of m. It turns out that, as in Krusell and Smith's case, the aggregate capital stock suffices to summarize the entire distribution of capital holdings. For the interested reader, we provide a detailed description of our solution method and some computational subtleties in Appendix A.

2.5 Full Insurance

Suppose that workers live together in a very large extended family, called a household. There are a continuum of identical households in the economy, and their mass is normalized to 1. The only difference with respect to the model outlined above is that each household member is perfectly insured by the other household members against variations in labor income due to changes in employment status. The existence of a market for Arrow securities among household members is the only difference with respect to the model presented in the previous section. The household's problem can then be written as the following dynamic programming problem:

$$J(z, K, N) = \max_{C,K'} U(C) + \beta \mathbb{E} J(z', K', N'),$$

$$s.t. \qquad C + K' = wN + b(1-N) + (R + 1 - \delta) K,$$

$$N' = N(1-s) + (1-N)\pi.$$

The firm's problem remains the same as before. Wages are determined by Nash bargaining. Hence the wage equation and the optimal job posting are given by:

$$w = (1-\alpha) z K^{\alpha} N^{-\alpha} + \frac{\omega(1-s)}{p} - \frac{\xi}{1-\xi} \frac{1}{u'(C)} \frac{\partial J(z,K,N)}{\partial N}, \tag{17}$$

$$\frac{\omega}{p} = \beta \mathbb{E}\left[\frac{\xi}{1-\xi} \frac{1}{u'(C')} \frac{\partial J(z',K',N')}{\partial N'}\right]. \tag{18}$$

where $\partial J(z, K, N)/\partial N$ is the marginal value of employment for a household and is defined by:

$$\frac{\partial J(z,K,N)}{\partial N} = u'(c) w' + (1 - s - \pi) \beta \mathbb{E}\left(\frac{\partial J(z',K',N')}{\partial N'}\right).$$

3 Parameterization

The model period is a month, as several parameters we calibrate are computed empirically using monthly data. In choosing functional forms and parameter values we have either followed previous research or set parameters to match a few steady state moments.

Regarding preferences we chose the constant relative risk aversion as our per period utility function. This functional form is widely popular in the macroeconomics literature and its only parameter is the relative risk aversion coefficient σ:

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

The value for σ that macroeconomists generally use, ranges from 1 to 4. We have chosen 1.5 as the benchmark but provide some sensitivity analysis for changing that value, see Section 4.2 for more details. The agents' discount factor β was set at 0.997.[7] This is the usual choice in infinite horizon economies modeled at the monthly frequency. In a complete markets framework it implies an annual interest rate of approximately 4.2 percent. The benchmark borrowing limit is normalized to 0.

The firm faces a Cobb-Douglas technology on capital and labor for producing output:

$$Y = zK^{\alpha}N^{1-\alpha}.$$

Evidence from the National Income and Product Accounts (NIPA) indicates that capital's share in national income has averaged about 36% for the US in the post-war period. Consequently, we set α to 0.36. The autocorrelation and the variance of the total factor productivity shock z_t are set to roughly match the observed persistence and variability of deviations from trend in the Solow residual. Denoting this residual by z_t, we take the quarterly AR(1) process fitted to post-war data by Silos (2007) and fit a two-state Markov chain to the logarithm of z_t. Given that our calibration is monthly, some care is needed so that the dynamics of the computed monthly process are consistent with the observed quarterly moments. At the quarterly frequency the standard deviation and first-order autocorrelation of $\log(z_t)$ are 2.05% and 0.983. In the monthly frequency it implies the same standard deviation and a first-order autocorrelation of 0.977. We therefore restrict z_t to take on two values: 1.0205 and 0.9795. The matrix that determines the rate of transition from expansions to recessions and vice versa is,

$$\Pi = \begin{bmatrix} 0.983 & 0.017 \\ 0.017 & 0.983 \end{bmatrix}. \tag{19}$$

We chose a Cobb-Douglas as the functional form for the matching technology. This

[7]See, for example, Shimer (2005) set the same value of beta at the monthly frequency.

is the most common choice in models of search and matching in labor markets.

$$M(V, 1-N) = \mu V^{\gamma}(1-N)^{1-\gamma}.$$

The parameter γ was set equal to ξ, the parameter driving the firm's bargaining power, which in complete markets models ensures that the allocation in the decentralized economy is the same as in the social optimum. Both were set at a value of 0.28, which we take from Shimer (2005). The parameters δ, μ, ω and b were set so that they match four moments in the data: an average job-finding probability of 0.45, an average vacancy-filling probability of 0.80, a vacancy-cost-to-output ratio of about 0.01, and a ratio of unemployment benefits to average wages of 0.42. There is little evidence on aggregate expenditures in recruiting. Andolfatto (1996) claims they are small and therefore sets the average vacancy-cost-to-output ratio to 0.01; we have also used that number. Shimer (2005) finds an average monthly job finding probability to be 0.45 using monthly gross worker flow data. The vacancy-filling rate is consistent with evidence presented in Blanchard and Diamond (1989), who find that vacancy postings have an average of three weeks, implying a vacancy-filing rate of 0.80. The home production parameter, b, is subject to much debate in the literature. Standard literature which follows Shimer (2005) views the value of "home production" to be commensurate with the average level of unemployment insurance which is roughly 42% of the wage. An alternative calibration, due to Hagedorn and Manovskii (2008), considers home production to be only slightly below the market wage which will lead to a much rigid wage. We consider this alternative parameter setting in Section 4.2.

Finally, the separation rate s was set at 0.035, a number used in previous studies of labor search and business cycles (e.g. Fujita and Ramey (2007)) and calculated in Abowd and Zellner (1985). Table 1 summarizes the parameterization.

Table 1: Summary of Parameterization

Parameter	Value	Target/Source
α	0.36	NIPA
β	0.997	$r \simeq 4.2\%$
s	0.035	Andolfatto (1996)
σ	1.5	–
ξ	0.280	Shimer (2005)
b	–	$b/w = 0.42$
μ	–	$wV/Y = 0.01$
δ	–	$q = 0.80$
ω	–	$\pi = 0.45$

4 Quantitative Results

4.1 Benchmark and Experiments

We focus on the cyclical dynamics of aggregate variables, especially the standard deviations relative to output. Table 2 shows the main result.[8] The first row displays statistics for the U.S. economy, while the remaining five rows show results for the five different models we consider: full insurance, baseline incomplete markets, stochastic-β, irreversible investment, and idiosyncratic earnings. The variables we have focused on are output, consumption, employment, the vacancy-unemployment ratio, and wages. All variables (except the employment rate and the vacancy-unemployment ratio) are in per capita real terms. Data on the job finding rate and the level of vacancies come from Robert Shimer's website. Data on consumption, output, corporate profits and wages are from the Bureau of Economic Analysis (National Accounts). The employment rate is defined as 1 minus the unemployment rate as reported by the Bureau of Labor Statistics. The sample period is 1951:1-2004:4 and all variables were logged and HP-filtered with a smoothing parameter of 1600. As the model period is a month, we aggregate the simulated data from our artificial economies by taking 3-month averages. We filter these series in the same fashion as we filter the US data.

Aside from the standard smaller volatilities of consumption and labor relative to output, the most noticeable feature of the data is the high volatility of the vacancy-

[8] Other business cycle properties such as autocorrelations, cross-correlations etc., are available upon request.

unemployment ratio with respect to GDP: it is larger by a factor of 17. In contrast, wages are less volatile than output. Consistent with the real business cycle literature, consumption, employment, the vacancy-unemployment ratio and wages are all quite procyclical, and employment also lags output slightly.

We start from the baseline incomplete markets model since its wealth distribution will serve as the benchmark for our analysis. As is clear from Table 2, when comparing the baseline economy with the full insurance economy, both economies are virtually indistinguishable. The volatilities of employment, the vacancy-unemployment ratio and wages are almost exactly the same. There is a difference in the volatility of consumption that decreases from 73% of that of GDP in the full insurance case to 55% in the uninsurable risk economy. Although in principle the mechanism outlined before makes wages somewhat smoother in the uninsurable risk economy, quantitatively the effect is negligible. The reason for the small difference is that agents overcome quite easily the lack of perfect insurance. Although they only have one asset, physical capital, to smooth out adverse shocks, the degree of persistence of the unemployment state is not too large and agents can smooth consumption quite easily. This results in a very similar behavior across the two economies. More evidence can be obtained by looking at the wealth distribution that results from the baseline uninsurable risk economy. Figure 1 shows the cumulative distribution of capital holdings for the baseline uninsurable risk economy: the fraction of agents close to the borrowing constraint is practically zero.

In an attempt to obtain a larger dispersion of wealth holdings, and in particular a larger fraction of agents close to the borrowing constraint, we separately introduce several elements that increase heterogeneity or limit self-insurance. We begin by allowing the discount factor to change stochastically. Instead of fixing β at 0.997, agents can transit across three degrees of patience: $\beta_L = 0.9952$, $\beta_M = 0.9964$ and β_H=0.9976. The transition matrix that determines the conditional probabilities for these two βs is:

$$\Pi_\beta = \begin{bmatrix} 0.998 & 0.002 & 0 \\ 0.0002 & 0.9996 & 0.0002 \\ 0 & 0.002 & 0.998 \end{bmatrix}. \tag{20}$$

In the matrix Π_β the first row shows the conditional probabilities of transiting or staying at the low patience state. The first element is the probability of remaining in the low patience state, and the second and third elements are the probabilities of switching to the medium and high patience states respectively. Analogously, the second and third rows display the conditional probabilities of staying and moving from the medium and high patience states respectively. The possible values for the discount factor and the transition probabilities are the monthly equivalents of those used in Krusell and Smith (1998). They interpret transitions between discounts factor as representing changes in generations in an economy populated by infinitely-lived agents. The wealth distribution for this economy is shown in Figure 2. When comparing to the baseline economy, the fraction of agents close to the borrowing constraint is clearly larger. The persistence of the β-shock couple with unemployment spells, causes impatient agents to deplete assets relatively quickly. The volatility of the vacancy-unemployment ratio and employment roughly double relative to the benchmark uninsurable risk and full insurance economies, and wages are about 4% smoother. Again, this is a consequence of the mechanism outlined before: agents with low degrees of patience have lower wealth holdings. This increases the fraction of agents close to the borrowing constraint and smooths wages relative to the perfect insurance economy.

Returning to the case of a single discount factor, we now add an irreversibility constraint at the individual level. This constraint limits the ability of agents to smooth consumption by limiting the amount of capital selling an individual can undertake in the face of an adverse employment shock. Formally, the constraint is written as:

$$k' \geq k(1-\delta) \tag{21}$$

We display the results for this case on the fifth row of the tables. The quantitative impact of the irreversibility constraint is almost negligible. We observe a slightly higher volatility of the vacancy-unemployment ratio (1.76 vs. 1.67 in the baseline model) and no change in the volatility of wages (0.96 relative to that of output). The cross correlations with output and the persistence of the macroeconomic series are quantitatively very similar to the ones obtained for the baseline idiosyncratic risk economy. One can safely conclude that the irreversibility constraint is easily overcome by the agents' savings behavior. Figure 3 shows exactly this point: the wealth distribution in the irreversible investment case comes close to the benchmark case comparing to Figure 2.

Finally, we increased the volatility of earnings by adding uncertain productivity levels for working agents. We denote this productivity shock by ϵ. This transforms the budget constraint for the worker to:

$$c^e + k^{e'} = w\epsilon + Rk + (1-\delta)\,k \tag{22}$$

The idiosyncratic productivity process is a finite-state approximation of the model for the idiosyncratic component of labor earnings estimated in Storesletten *et al.* (2004). Their sample is annual covering the period 1968-1993, with data from the Panel Study of Income Dynamics (PSID). Denoting by $u_{it} = \log(y_{it})$ the logarithm of the idiosyncratic component of labor income for household i at time t, the model estimated is:

$$u_{it} = z_{it} + \epsilon_{it} \tag{23}$$

$$z_{it} = \rho z_{i,t-1} + \nu_{it}$$

where $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$ and $\nu_{it} \sim N(0, \sigma_\nu^2)$. Fernandez-Villaverde and Krueger (2007) report $\rho = 0.935$, $\sigma_\epsilon^2 = 0.017$ and $\sigma_\nu^2 = 0.061$. We have approximated this process as a two-state Markov Chain restricting our "monthly" transition matrix to be consistent with the estimated autocorrelation (at the annual frequency) from Storesletten *et al.* (2004). The resulting support for $y = e^u$ is the set $\{0.648, 1.352\}$ with transition probability matrix:

$$\Omega = \begin{bmatrix} 0.996 & 0.004 \\ 0.004 & 0.996 \end{bmatrix}. \tag{24}$$

The sixth row Table 2 displays the standard deviation for this economy. There is a quantitatively significant improvement with respect to the previous two modifications, and the volatility of the vacancy-unemployment ratio with respect to output almost triples, relative to the full insurance and the baseline idiosyncratic risk economies. The real wage is smoother, 0.87 relative to output, while in the full insurance and the baseline uninsured risk economies, the volatility of real wages was 0.96. This is a 9% decline in the volatility of wages relative to the value observed in the data, 0.68. The asset CDFs for this and the baseline uninsurable risk economies are shown in Figure 4. In the idiosyncratic productivity economy the wealth inequality is significantly larger, with a smaller fraction of agents holding "average levels of capital". The differences are also appreciable relative to stochastic-β economy. The fraction of agents within with capital holdings equal or less than 10% of average capital is close to 20%.

4.2 Sensitivity Analysis

In this section, we provide some sensitivity analysis to check whether the presented business cycle dynamics in the baseline economy remain unchanged. We first change the the curvature of the utility function. While keeping the other parameters constant, we increase σ from 1.5 to 3.5. As agents become more risk averse, they will accumulate more assets to smooth consumption. As a result, the average capital holding increases from 45.73 to 46.55, which in turn causes some level differences in consumption, investment and wages. In terms of volatilities, increase in the risk aversion only has a very mild effect on labor market dynamics: the vacancy-unemployment ratio rises from 1.67 to 1.70 as Table 3 shows, even though fluctuation in employment changes from 0.03 to 0.05 which is still far from the data. The volatility of wage is similar between these two parameter values. Given the same ability to smooth out consumption, consumption becomes more volatile if agents become more risk averse.

The second parameter that we change is the home production parameter b. Instead of targeting the unemployment benefit to wage ratio (b/w) as 0.42, we follow the spirit of Hagedorn and Manovskii (2008) to increase it to 0.9. Since the unemployment benefit is close to the average wage, the wage will become more rigid, for example, in response to a negative shock, w cannot drop too much to below b, and thus, the firm has to reduce job postings and cut positions. This is the mechanism that the HM calibration can generate a very large fluctuation in unemployment and vacancies. The last row of Table 3 shows the results: wage becomes less volatile while the employment and vacancies become much more volatile. These findings are consistent with the points made by Krusell *et al.* (2010). Given the high unemployment benefit, the wealth distribution becomes less dispersed as Figure 5 illustrates. People can insure against idiosyncratic risks more easily if they have more generous unemployment benefits. The lower wealth dispersion can partly explain why the model still falls short to match the observed volatility of vacancy-unemployment ratio (6 vs 17).

4.3 The Cyclical Dynamics of the Labor's Share

The "endogenous" wage rigidity that our mechanism delivers, brings the volatility of the vacancy-unemployment ratio closer to the data. However, as pointed out by Hornstein *et al.* (2005), wage rigidity introduces a whole set of new problems. We want to focus here on the cyclical behavior of labor's share, which equals $LSH_t = \frac{w_t N_t}{Y_t}$. In US data, the contemporaneous correlation of the labor's share with output $\rho(Y_t, LSH_t)$ is -0.14, which is somewhere between acyclical and mildly counter-cyclical. In our baseline uninsurable risk model, this correlation is -0.50. As wage rigidity rises, the volatility of the vacancy-unemployment ratio also rises, but the correlation between labor's share and output falls. In our model economy with idiosyncratic productivity shocks, the volatility of the vacancy-unemployment ratio triples, but $\rho(Y_t, LSH_t)$ becomes -0.98. This correlation is too extreme compared to the data and it is an unintended consequence of wage rigidity. On a more positive note however, the labor's share is too smooth in the baseline uninsurable risk economy; its volatility relative to output is 0.022 (0.37 in the data). This volatility increases to 0.05 in the idiosyncratic productivity shocks economy, which is still far from its empirical value, but closer. We refer the interested reader to Shao and Silos (2013) and Shao and Silos (2014) for a deeper investigation of the joint cyclical dynamics of profits' and labor's share in a search and matching framework.

5 Conclusion

The attitude towards risk and the absence of perfect insurance is an assumption that is missing from many studies of economic fluctuations with search in the labor markets. Our research shows that the heterogeneity in asset holdings that results from assuming imperfect insurance of idiosyncratic earnings risk acts as a mechanism that decreases the volatility of wages and increases the volatility of the vacancy-unemployment ratio. The reason is that when negotiating wages, the fraction of poor workers accept lower wages than they otherwise would. Our starting point has been the Mortensen-Pissarides economy, to which we have added idiosyncratic risk and limited the ability of agents to insure against that risk. We show how heterogeneity in asset holdings helps when bringing the model's implication closer to the data.

In our baseline parameterization with incomplete markets, although agents only have access to one asset to smooth consumption, the degree of self insurance is remarkably good. The Mortensen-Pissarides economy where agents are unable to perfectly insure against the risk of being separated from their current job or not being matched with a firm, is virtually indistinguishable from the complete markets economy. To obtain sizable differences in the volatility of the vacancy-unemployment ratio one needs to obtain high wealth inequality. Two features that we have explored here are, first, to assume that agents have varying degrees of patience, which affect the preferred rate of asset accumulation; and second, to assume a large dispersion in productivity within working agents. Each of these two features increases substantially the standard deviation of the vacancy-unemployment ratio relative to output.

A Solution Algorithm

The relevant aggregate state variables in the individual's problem is $\tilde{\chi} = (z, N, K)$. Notice that we have already replaced the distribution Q by its first moment K. As we show below, the accuracy of projecting K', w and π on just K and N is extremely good. The solution of the model entails computing the following objects.

1. Optimal decision rules for consumption $c^e(k; \tilde{\chi})$ and $c^u(k; \tilde{\chi})$, the value function $J(k; \tilde{\chi})$ and the marginal value of employment $(\partial J/\partial N)(k; \tilde{\chi})$,

2. a matching probability function $\pi(\tilde{\chi})$,

3. a wage function $w(\tilde{\chi})$

4. the law of motion for aggregate capital $K'(\tilde{\chi})$.

A.1 Overview of the Algorithm

The solution algorithm is made up of the following steps (we will fill in the details in later subsection):

1. Choose aggregate grid points for N and K and the individual asset grid k.

2. Choose the class of polynomials to approximate the aggregate law of motion $K'(\tilde{\chi})$, the job finding rate $\pi(\tilde{\chi})$ and the wage function $w(\tilde{\chi})$. Make an initial guess on the coefficients of above functions. Choose suitable interpolation schemes to approximate the consumption functions $c^e(k; \chi)$ and $c^u(k; \tilde{\chi})$, the decision rules $k'_e(k; \tilde{\chi})$ and $k'_u(k; \tilde{\chi})$, and the value functions $J(k; \tilde{\chi})$ and $(\partial J/\partial N)(k; \tilde{\chi})$.

3. For a given aggregate law of motion, job finding probability and wage rate, solve for the workers problem. This step involves solving for c^e, c^u, k'_e, k'_u, J and $\partial J/\partial N$ at each grid point.

4. Given an initial guess on the wealth distribution, simulate the economy for a long time series and use the policy rules obtained in (3) to calculate the wealth distribution Q, the matching probability π and the wage rate for each period. This step involves iteratively solving for the optimal job posting equation (15).

5. Use the stationary region of the simulated data to estimate the new coefficients in $K'(\tilde{\chi})$, $\pi(\tilde{\chi})$, and $w(\tilde{\chi})$.

6. Repeat steps 3-5 until convergence of the relevant functions is achieved.

7. Check whether the goodness of fit is satisfactory. It it is not, then increase the moments used to approximate the wealth distribution or try a different functional forms for K', π and w.

A.2 Detail Description of the Algorithm

A.2.1 Solving the worker's optimization problem

1. Setup the grid on k', the end of period capital holdings (or next period capital holdings). The grid of points is $\{k'_1, ..., k'_n\}$ with $k'_1 = \underline{k}$ the borrowing limit. Usually this grid is finer than the asset grid k.

2. Initially assume that workers do not save for tomorrow, which means they will consume all the income:

$$\begin{aligned} c_0^e &= \left(R\left(\tilde{\chi}\right) + 1 - \delta\right) k + w\left(\tilde{\chi}\right), \\ c_0^u &= \left(R\left(\tilde{\chi}\right) + 1 - \delta\right) k + b. \end{aligned}$$

Then calculate the value functions

$$\begin{aligned} J_0 &= NU\left(c_0^e\right) + \left(1 - N\right) U\left(c_0^u\right), \\ \left(\frac{\partial J}{\partial N}\right)_0 &= U\left(c_0^e\right) - U\left(c_0^u\right). \end{aligned}$$

3. At iteration step $t \geq 1$, given any approximation of policy functions c_{t-1}^e and c_{t-1}^u, calculate next period marginal utilities of consumption at each grid point

$(k_i', \tilde{\chi})$:

$$
\begin{aligned}
MU^e(k_i', \tilde{\chi}) &= \sum_{z'} p(z'|z) R(\tilde{\chi}') \times \\
&\quad N'(\tilde{\chi}) U'\left(c_{t-1}^e(k_i', \tilde{\chi}')\right), \\
MU^u(k_i', \tilde{\chi}) &= \sum_{z'} p(z'|z) R(\tilde{\chi}') \times \\
&\quad (1 - N'(\tilde{\chi})) U'\left(c_{t-1}^u(k_i', \tilde{\chi}')\right),
\end{aligned}
$$

where $N'(\tilde{\chi}) = (1-s)N + \pi(\tilde{\chi})N$.

4. From the Euler equations

$$
u'\left(\widetilde{c}^j\right) = \beta\left(MU^e(k_i', \tilde{\chi}) + MU^u(k_i', \tilde{\chi})\right), \tag{25}
$$

we can calculate the current consumption $(\widetilde{c}_i^e, \widetilde{c}_i^u)$ for each grid points k_i', $i = 1, ..., n$.

5. Use the budget constraints to recover the market resources (or income) at the beginning of current period

$$
\widetilde{y}_i = \widetilde{c}_i + k_i'
$$

6. Then $\{\widetilde{y}_i\}_{i=1}^n$ forms an endogenous grid on current income. Based on the set of pairs $\{(\widetilde{y}_i, \widetilde{c}_i)\}_{i=1}^n$, because

$$
\begin{aligned}
y^e &= (R(\tilde{\chi}) + 1 - \delta) k + w(\tilde{\chi}), \\
y^u &= (R(\tilde{\chi}) + 1 - \delta) k + b,
\end{aligned}
$$

we can simply use linear interpolation or other shape preserving schemes to obtain the policy functions $\hat{c}^j(y^j, \tilde{\chi})$ for given values of aggregate states $(\tilde{\chi})$. We can update the optimal consumption $c_t^j(k, \tilde{\chi})$ from $\hat{c}^j(y^j, \tilde{\chi})$.[9]

[9] To handle the borrowing constraints $k_j^{'} \geq \underline{k}$, $j = e, u$, we need to do the following. If for any given values of $(k, \tilde{\chi})$, $y^j \leq \widetilde{y}_1$, it implies that the borrowing constraint binds, we set $c_t^j(k, \tilde{\chi}) = y^j - \underline{k}$ and $k_j'(k, \tilde{\chi}) = \underline{k}$.

7. Given those values computed in (6), we then interpolate c_t^e and c_t^u among aggregate states (z, N, m).

8. Once we have the optimal consumption (c_t^e, c_t^u) and the value function J_{t-1}, we compute the new value function

$$\begin{aligned} J_t(k,z,N,m) &= N\begin{bmatrix} U\left(c_t^e(k,\tilde{\chi})\right) \\ +\beta\sum_{z'} p(z'|z)\, J_{t-1}(k_e', \tilde{\chi}') \end{bmatrix} \\ &\quad + (1-N)\begin{bmatrix} U\left(c_t^u(k,\tilde{\chi})\right) \\ +\beta\sum_{z'} p(z'|z)\, J_{t-1}(k_u', \tilde{\chi}') \end{bmatrix}, \end{aligned}$$

where k_j' can be calculated from

$$\begin{aligned} k_e'(k,\tilde{\chi}) &= \left(R(\tilde{\chi}) + 1 - \delta\right)k + w(\tilde{\chi}) - c_t^e(k,\tilde{\chi}), \\ k_u'(k,\tilde{\chi}) &= \left(R(\tilde{\chi}) + 1 - \delta\right)k + b - c_t^u(k,\tilde{\chi}). \end{aligned}$$

9. Use (c_t^e, c_t^u, J_t) and $(\partial J/\partial N)_{t-1}$ to update the new marginal value of employment:

$$\left(\frac{\partial J}{\partial N}\right)_t(k,z,N,m) = U(c_t^e) - U(c_t^u) + \beta\sum_{z'} p(z'|z)\begin{pmatrix} J(k_e', \tilde{\chi}') \\ -J(k_u', \tilde{\chi}') \end{pmatrix}$$
$$+ \left(1 - s - \pi(\tilde{\chi})\right)\begin{bmatrix} N\beta\sum_{z'} p(z'|z)\left(\frac{\partial J}{\partial N}\right)_{t-1}(k_e', \tilde{\chi}') \\ +(1-N)\beta\sum_{z'} p(z'|z)\left(\frac{\partial J}{\partial N}\right)_{t-1}(k_u', \tilde{\chi}') \end{bmatrix}.$$

10. Repeat steps (3)-(9) until c^e, c^u, J, $\partial J/\partial N$ converge.

Since we solve the model on a discrete grid of points, the policy functions and value functions that we describe in the above steps have to be approximated between grid points. A good interpolation method that preserves the monotonicity and concavity of the value function is crucial for the stability and accuracy of the algorithm. Most Chebychev polynomial basis interpolation or other higher order approximations, including many forms of splines, can destroy the stability of the algorithm by producing internodal oscillations. For the sake of stability, we use the simplicial linear interpolation described in Judd (1998) which preserves the contraction property of the Bellman operator, which guarantees convergence. Since the dimension is less than 4, the simplicial linear interpolation is relatively easy to implement in our application. We setup the grid in k and k' direction so that we include many points near the borrowing limits (where there is a lot of curvature) and few grid points for larger values. The number of points are 50-60 for k and 150-200 for k'. Our results are not sensitive to increasing the number of grid points in either the k or k' direction.

A.2.2 Computation of the wealth distribution

One of the main steps in solving the model is to pin down the law of motion K'. In order to calculate it, we need to derive a time series of aggregate capital stocks $\{K_t\}_{t=1}^{T}$ and use this time series to estimate the transition function H mapping K_{t+1} into K_t. One possible approach to generate K_t is to simulate the behavior of a large number of consumers for each time period as proposed in Krusell and Smith (1998) and compute K_t as the average of their holdings. The drawback of this simulation method is that it is inaccurate, even with a very large number of agents. Here we discretize the state space and approximate the CDF as a step function to avoid doing any Monte Carlo simulation. The computation can be summarized as follows:

1. Simulate a long time series of aggregate shocks of length T using the transition matrix (20).

2. Specify grids on individual capital holdings k such that the grid is finer than the one used to compute the optimal decision rules. We use 240 to 400 grid points in this step.

3. Choose an initial distribution function $Q_0(k)$ over the grid. We generally assume that everyone has the same capital stock to begin with. We also try other distribution function such as uniform distribution, but it won't affect the result.

4. Use the decision rules calculated from section A.2.1, we can compute the inverse of the decision rules $k_i^j = k_j'^{-1}(k_i, \tilde{\chi})$, $j = e, u$, over the chosen grid.

5. Given the distribution Q_n and aggregate values $(\tilde{\chi})$ at time period n, the distribution at $n+1$ is

$$Q_{n+1}(k_i) = NQ_n\left(k_e'^{-1}(k_i, \tilde{\chi})\right) + (1-N)\,Q_n\left(k_u'^{-1}(k_i, \tilde{\chi})\right)$$

on grid points k_i. For those points $k_i^j = k_j'^{-1}(k_i, \tilde{\chi})$ are not grid points, we use linear interpolation to calculate $Q_n\left(k_i^j\right)$.

6. Compute the aggregate moments at time $n+1$ using Q_{n+1}. For example the aggregate capital is given by

$$K_{n+1} = k_1 Q_{n+1}(k_1) + \sum_{i=2}^{\eta_k} \left(Q_{n+1}(k_i) - Q_{n+1}(k_{i-1})\right)(k_i + k_{i-1})/2.$$

   where η_k is the number of grid points in k for the purpose of computing the wealth distribution. The dimension of this grid should be in general larger than the dimension of the grid to compute the decision rules.

7. After getting the long time series for aggregate capital, we can run the regressions to compute the law of motion for K' and the π and w functions.

A.2.3 Solving the optimal job posting

To find the wage and the matching probability, it is necessary to solve for the optimal vacancies in equation (15). Notice that (15) is a nonlinear in V, which appears in both hand sides of the equation.[10] We may use nonlinear equation solver to solve for V, however, it is easy to fail in getting the solution. We use similar idea of solving the worker's problem to iteratively find the fixed point of V.

Along the simulation path, for any period of time n, we are given the value of aggregate states $(\tilde{\chi})$. (1) We start with an initial guess on V, then we calculate the next period employment $N'$and the left-hand side of equation (15). (2) Use the procedure in section A.2.2 to compute the next period wealth distribution and update the aggregate moments for the next period. (3) Base on states $(\tilde{\chi}')$ and distribution Q', calculate the right-hand side of equation (15) using the functions from section (A.2.1). (4) If the difference between both hand side of the equation is smaller than the tolerance value , stop; otherwise repeat steps (1) - (3).

As one can see, the above iterative procedure is embedded into the computation of the wealth distribution. Once we solve for V, we can calculate w and π for any

[10]The left-hand side can be written as $\frac{\omega}{\mu}\left(\frac{V}{1-N}\right)^{1-\gamma}$. On the right-hand side, the function $\partial J(k', \tilde{\chi}')/\partial N'$ is a function of N' which in turn implicitly depends on V.

particular time period.

A.3 Numerical Solution

Table 4 documents some details about the numerical solutions. In choosing the grid points for individual capital, the borrowing constraint provides the lower bound for k. The upper bound of k is set to be 3 - 4 times larger than the steady state value of aggregate capital in the full insurance case. Unfortunately, there is no much guidance available when specifying the grids for the aggregate states. Finding sensible bounds required substantial trial and error. We chose a log-linear form for the law of motion of K' and for w and π. The coefficients in these functions are obtained by running OLS regressions. We report the equilibrium results in Tables 5-6. We can see that the measures of fit, either the R^2 or the relative errors [11], are extremely good, showing that increasing the moments in the wealth distribution would bring marginal gains.

[11] We report the R^2s of these regressions as it is customary in the literature. We are, however, aware of the criticisms made by Den Haan (2010) for assessing the accuracy of the aggregate laws of motion using this statistic.

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Table 2: Standard Deviations (Relative to Output)

Model	$\frac{\sigma_N}{\sigma_Y}$	$\frac{\sigma_C}{\sigma_Y}$	$\frac{\sigma_{VU}}{\sigma_Y}$	$\frac{\sigma_{wage}}{\sigma_Y}$
US Data	0.49	0.53	16.58	0.68
Baseline	0.03	0.55	1.67	0.96
Full Ins.	0.03	0.73	1.62	0.96
Stochastic-β	0.06	0.53	3.01	0.92
Irr. Invmt.	0.04	0.52	1.76	0.96
Idio. Earn.	0.09	0.50	4.31	0.87

Table 3: Sensitivity Analysis: Standard Deviations (Relative to Output)

Model	$\frac{\sigma_N}{\sigma_Y}$	$\frac{\sigma_C}{\sigma_Y}$	$\frac{\sigma_{VU}}{\sigma_Y}$	$\frac{\sigma_{wage}}{\sigma_Y}$
US Data	0.49	0.53	16.58	0.68
Baseline	0.03	0.55	1.67	0.96
CRRA Coefficient = 3.5	0.05	0.64	1.70	0.98
$b/w = 0.9$	0.71	0.69	6.05	0.78

Table 4: Details of numerical solutions

Property	Benchmark	Full Insurance	Irreversible Investment
Moments Used	Mean	Mean	Mean
Interpolation Method	Piecewise Linear	Piecewise Linear	Piecewise Linear
Grid Dimension			
Individual Problem	$\eta_k = 50$, $\eta_{k'} = 150$	N/A	$\eta_k = 55$, $\eta_{k'} = 200$
Aggregate States	$\eta_N = 5$, $\eta_K = 5$	$\eta_N = 10$, $\eta_K = 50$	$\eta_N = 5$, $\eta_K = 5$
Wealth Distribution	$\eta_k = 240$	N/A	$\eta_k = 350$
Functional Form	Log Linear	Log Linear	Log Linear

Table 5: Equilibrium Results of Benchmark Model

Function	Coefficients	R^2	Relative Errors
H_I	$\ln K' = 0.109 + 0.053 \ln z + 0.127 \ln N + 0.975 \ln K$	1.0	0.01%
$w(zh)$	$\ln w = -0.195 + 1.294 \ln N + 0.328 \ln K$	1.0	0.02%
$w(zl)$	$\ln w = -0.218 + 1.375 \ln N + 0.326 \ln K$	1.0	0.02%
$\pi(zh)$	$\ln \pi = -1.312 + 1.839 \ln N + 0.164 \ln K$	1.0	0.02%
$\pi(zl)$	$\ln \pi = -1.321 + 1.961 \ln N + 0.164 \ln K$	1.0	0.02%

Table 6: Equilibrium Results of Irreversible Investment

Function	Coefficients	R^2	Relative Errors
H_I	$\ln K' = 0.096 + 0.048 \ln z + 0.049 \ln N + 0.975 \ln K$	1.0	0.01%
$w(zh)$	$\ln w = -0.424 - 0.367 \ln N + 0.353 \ln K$	1.0	0.05%
$w(zl)$	$\ln w = -0.470 - 0.358 \ln N + 0.354 \ln K$	1.0	0.05%
$\pi(zh)$	$\ln \pi = -1.536 + 0.178 \ln N + 0.184 \ln K$	1.0	0.02%
$\pi(zl)$	$\ln \pi = -1.613 + 0.008 \ln N + 0.196 \ln K$	1.0	0.02%

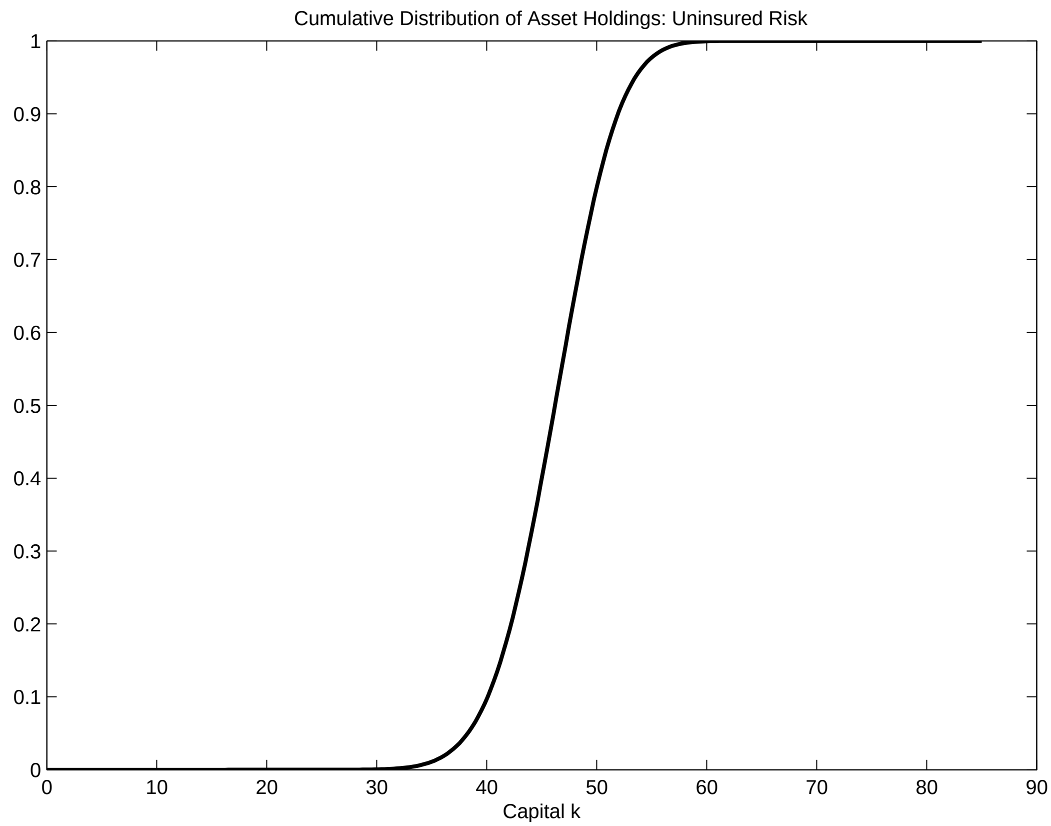

Figure 1: CDF of asset holdings in the baseline uninsurable risk economy.

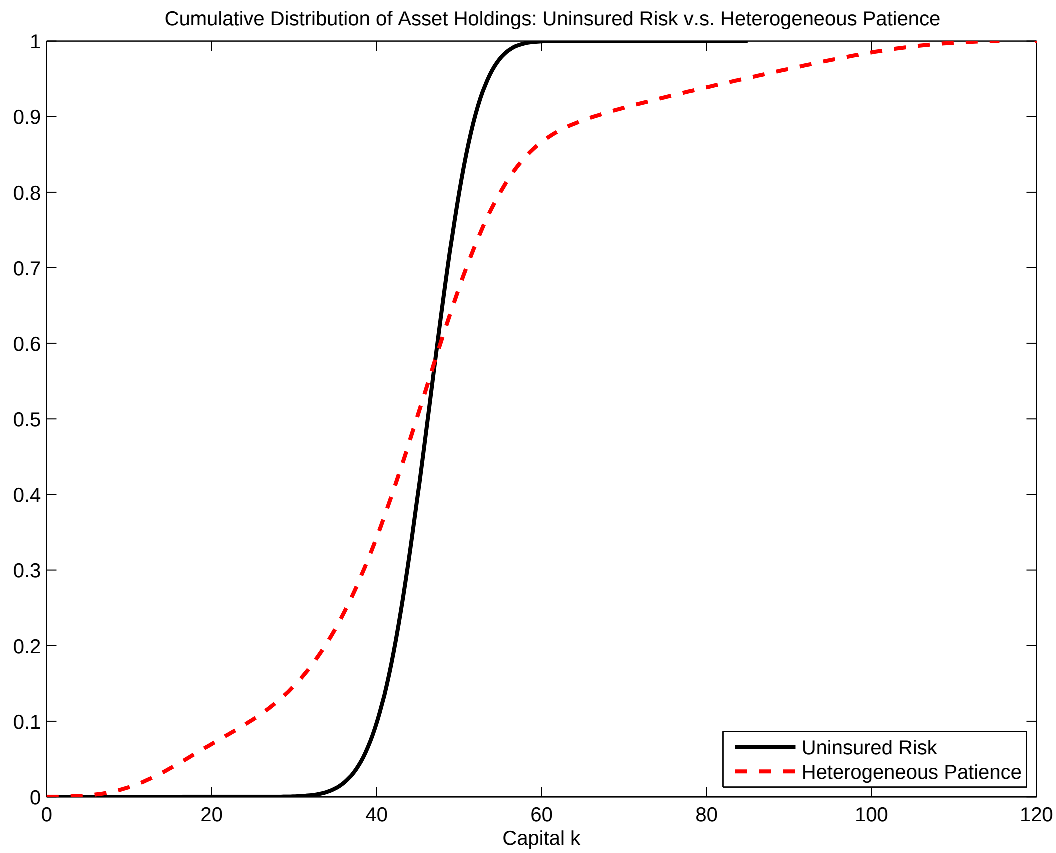

Figure 2: CDF of asset holdings in the economy with varying discount factors vs the baseline uninsurable risk economy.

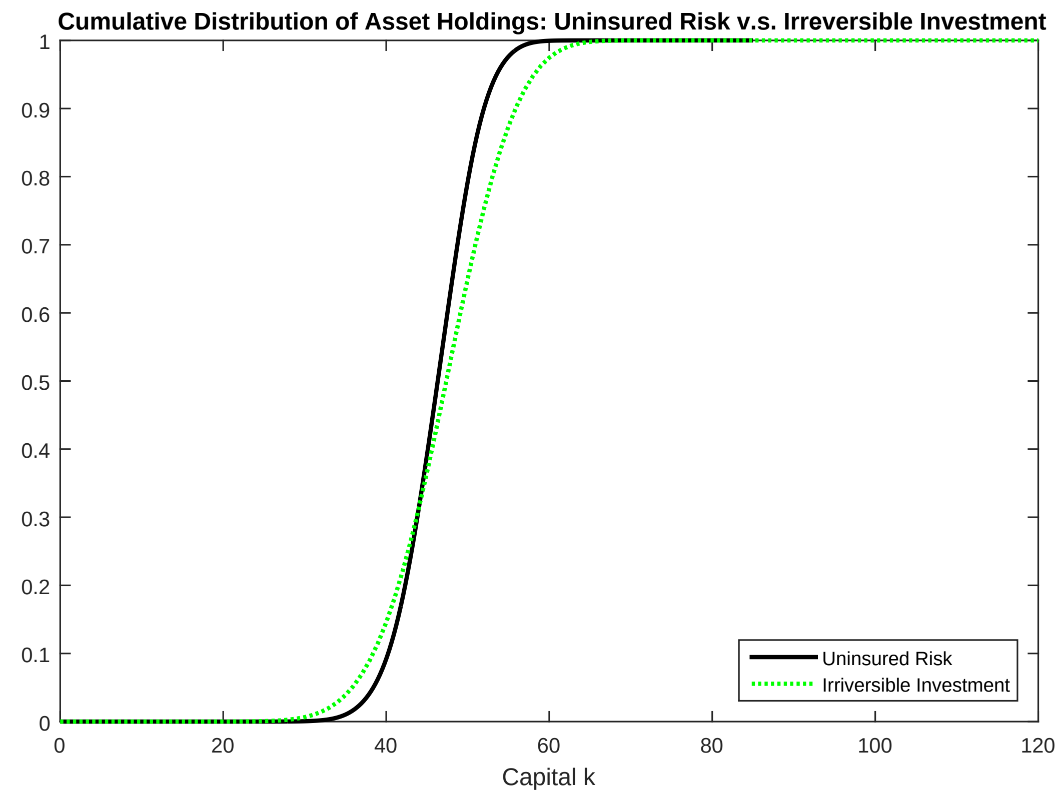

Figure 3: CDF of asset holdings in the economy with irreversible investment vs the baseline uninsurable risk economy.

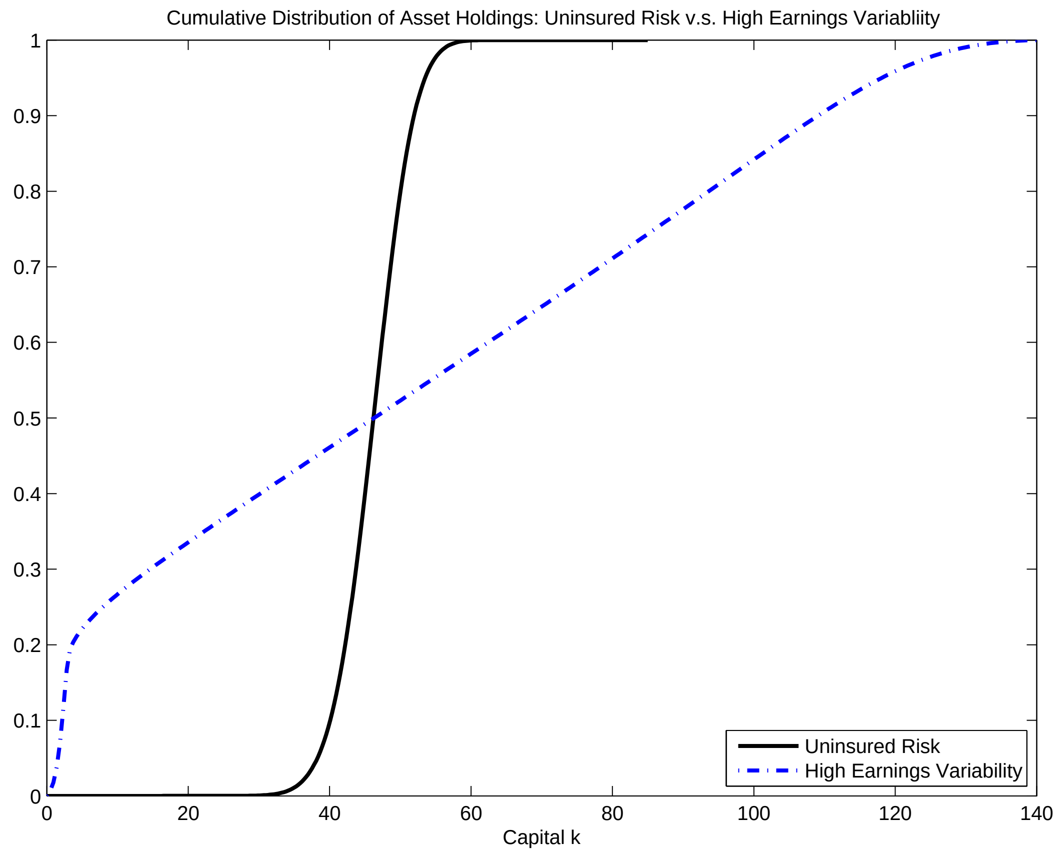

Figure 4: Comparison of the CDFs of asset holdings in the baseline uninsurable risk economy (solid line) versus the economy with idiosyncratic productivity shocks (dotted line).

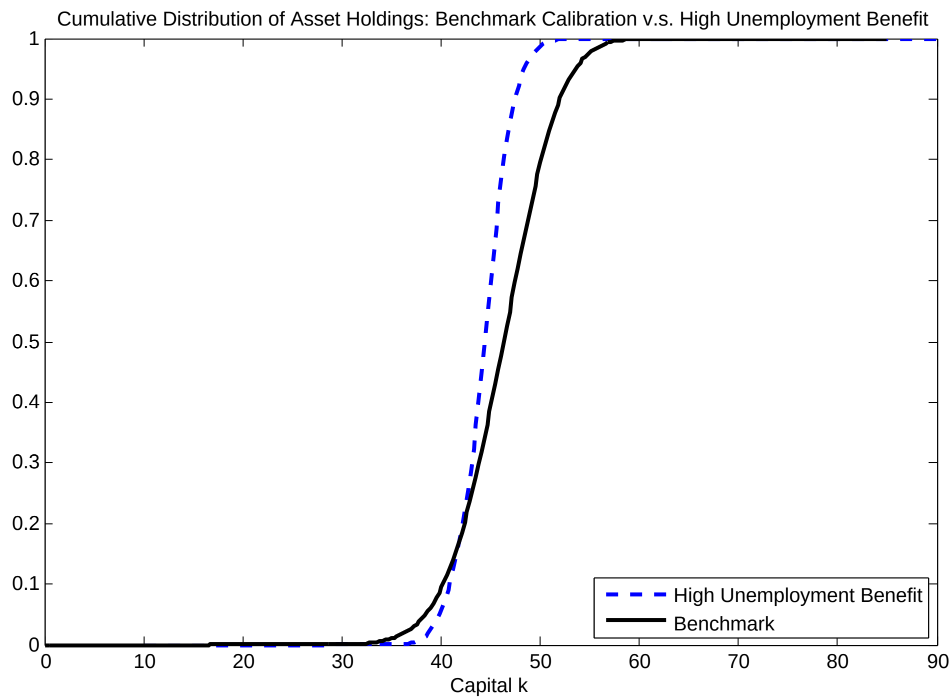

Figure 5: Comparison of the CDFs of asset holdings in the baseline calibration (solid line) versus the economy with high unemployment benefit (dotted line).