

# Accounting for the Cyclical Dynamics of Income Shares\*

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## Abstract

Over the business cycle, labor's share of output is negatively but weakly correlated with output, and it lags output by about four quarters. Profits' share is strongly pro-cyclical. It neither leads nor lags output, and its volatility is about five times that of output. Those assumptions relate to the structure of aggregate technology and the degree of competition in factor markets. Despite much evidence in favor of time-varying income shares, macroeconomics still lacks models that can account for their time series facts. This paper constructs a model that can replicate those facts. We introduce costly entry of firms in a model with frictional labor markets and find a link between the ability of the model to replicate income shares' dynamics and the ability of the model to amplify and propagate shocks. That link is a weak correlation between the real interest rate and output, a fact in US data but a feature that models of aggregate fluctuations have had difficulty achieving.

*JEL codes:* E3, E25, J3, E24

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# 1 Introduction

This paper constructs a framework to quantitatively account for the business cycle dynamics of profits' and labor's share. Despite the evidence in favor of the time-varying behavior of income shares, macroeconomics lacks models that can quantitatively match their time series facts. This paper reviews those facts and discusses the inability of existing models to replicate them. It then presents a model and shows that under a careful parameterization, it can quantitatively account for, if not all, many of the properties that describe the behavior of income shares.

Labor's share is defined as the share of national income that accrues to labor. Figure 1 displays the correlation of labor's share with real GDP at various leads and lags from 1951 to 2007.<sup>1</sup> Correlations are not strong, the maximum is about 0.50, and the contemporaneous correlation is (significantly) smaller than the correlation between output and the labor's share with a four-period lead. Consequently, labor's share lags real GDP because its correlation coefficient with output is highest after four quarters. Labor's share is counter-cyclical but very weakly so: the contemporaneous correlation is -0.11, and the 5th percentile for the sample distribution of that correlation is 0.02.<sup>2</sup> On the other hand, Figure 2 shows that the profits' share is strongly pro-cyclical and it neither leads nor lags output; its volatility is about 5.2 times that of output.<sup>3</sup>

As perfect competition in labor markets implies a tight link between wages and output, previous studies have deviated from a Walrasian labor market in an attempt to explain these facts. They have done so by specifying contractual arrangements between employers and employees that have broken the link between wages and the marginal product of labor. The goal was to match properties of labor's share over the business cycle. Examples of this line of work include [Boldrin and Horvath \(1995\)](#), [Gomme and Greenwood \(1995\)](#), and [Danthine and Donaldson \(1992\)](#). These authors can only qualitatively match the procyclicality of profits' and the counter-cyclicality of labor's share, but quantitatively their models' results are far

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<sup>1</sup>We focus on business cycle frequencies so all data are logged and HP filtered with a smoothing parameter equal to 1600. The statistics discussed in the introduction and shown in Figure 1 do not depend on how one defines labor's share. In the appendix we provide four alternative definitions and show how labor's share's business cycle dynamics are similar across those definitions.

<sup>2</sup>These facts have been reported elsewhere, with minor quantitative differences, most recently by [Rios-Rull and Santaaulalia-Llopis \(2010\)](#).

<sup>3</sup>Profits' share is not calculated as one minus labor's share: it is defined as the ratio of corporate profits to aggregate income. To be consistent with the model we construct, payments to capital are separated from payments to shareholders and we provide a discussion at the end of the introduction about separating those two concepts in the data.

from the data.

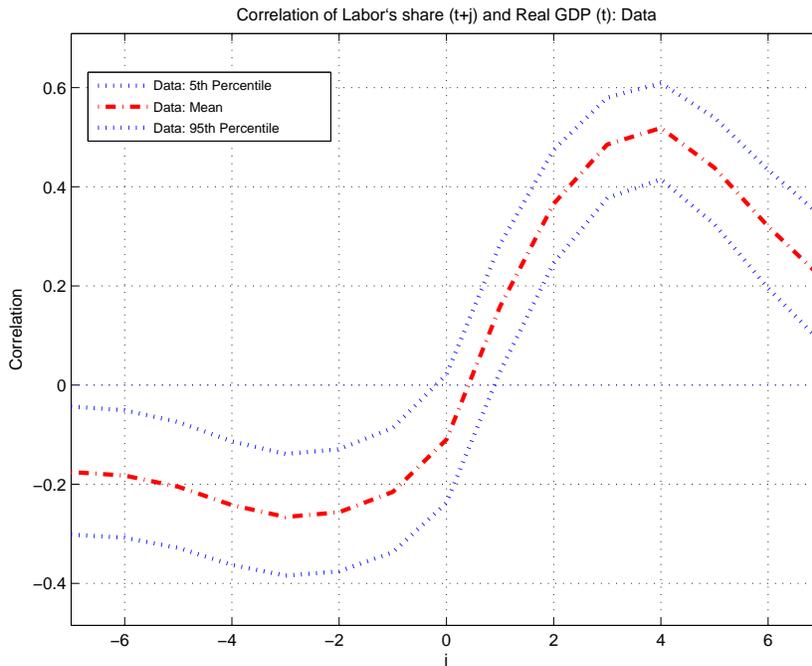


Figure 1: Correlations between Real Gross Domestic Product and (led and lagged) Labor's Share of Real GDP - US Data 1951-2007.

Dispensing with a Walrasian framework is a characteristic of literature that features search and matching frictions in labor markets, e.g. [Pissarides \(1985\)](#). Our research falls within this framework.<sup>4</sup> Although this literature has claimed success in matching some labor market business cycle moments, we show that they can't account quantitatively for the dynamics of income shares. We link this failure to the typical assumption of free entry of firms, which leads to the asset value of a vacant position to be exactly zero at all frequencies. That free entry implies that the value of a vacant position equals zero can be seen easily from the textbook model of search and matching, for example [Pissarides \(2000\)](#). If it were positive, firms would continue to post vacancies, lowering the probability that a given vacancy gets filled until its present value reached zero.

We construct an environment<sup>5</sup> in which the present value of a vacant position is always

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<sup>4</sup>An alternative approach to study time-varying income shares is to introduce that time variation exogenously. This exogeneity still allows the researcher to analyze joint dynamics of those income shares with endogenous variables; output, for example. This is done by [Young \(2004\)](#), who also shows that the contemporaneous correlation between output and the labor's share is close to -1.

<sup>5</sup>The environment described follows closely that in [Shao and Silos \(in press\)](#). The focus here is on time

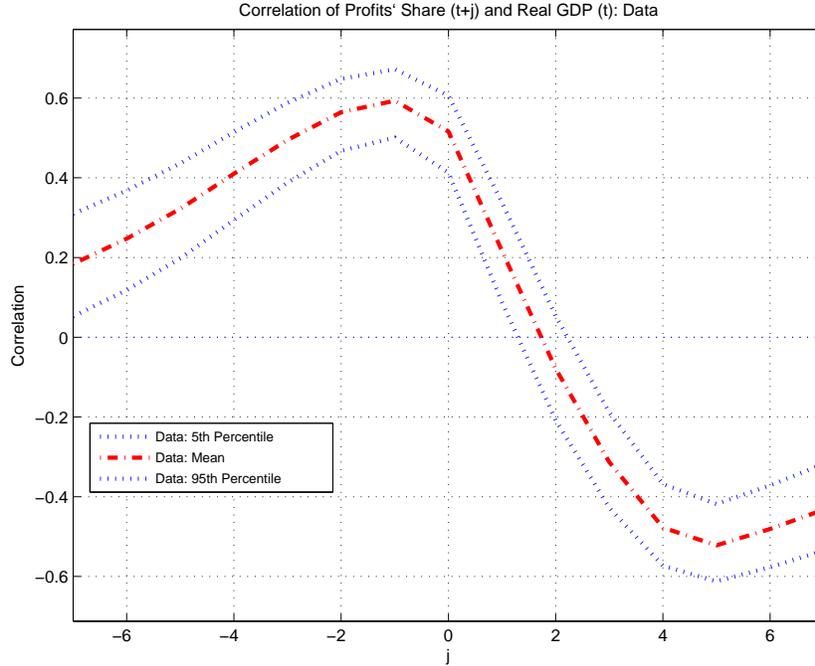


Figure 2: Correlations between Real Gross Domestic Product and (led and lagged) Profits' Share of Real GDP - US Data 1951-2007.

positive and endogenously varies over the business cycle. A vacancy has positive asset value because firms need to incur entry costs before they are allowed to post a vacancy, hire workers, and begin production. The equilibrium value of a vacancy is equal to the sunk cost, so that firms are indifferent between entering or staying out of the market. This equilibrium asset value is also time-varying. The reason: entrants rent factors of production to pay for the sunk cost and the efficiency of these factors is affected by the same shocks that generate aggregate fluctuations. As the prices and quantities of these factors vary with aggregate conditions, so do the expenditures that entrants undertake. In equilibrium, these expenditures must equal the capital value of a vacancy.

To some extent, our economy resembles a two-sector environment. The first sector produces goods and services that households consume, and the second sector produces services that entrants need to purchase. These purchases, in turn, allow entrants to access the goods-producing sector and profit from the sales of those goods. However, those two sectors compete for the same factors of production causing the dynamics of entrants to influence the dynamics of the demand, and hence prices, of those factors. Based on specific model-series behavior of labor's and profit's share in output in relation to the dynamics of the real interest rate.

ing assumptions described later in this paper, the behavior of these prices determines the dynamics of the value of a vacancy. We show that a reasonable parameterization of our environment can match the joint dynamics of labor's share, profits' share, and output. The model is consistent with the lagging behavior of labor's share and its weak correlation with output. The model is also consistent with the strong correlation of output with profits' share and its lack of leading or lagging behavior.

We show that these results depend on one equilibrium outcome: that the value of a vacancy is counter-cyclical. In fact, parameterizations of our economy that yield a procyclical capital value of a vacancy are inconsistent with the dynamics of income shares, and they feature virtually no amplification of shocks. Unfortunately, no good empirical data exist as counterparts to the asset value of a vacancy. Fortunately, in our model, the dynamics of the value of a vacancy are mainly driven by the dynamics of the real interest rate. However imperfect, we do have measures of real interest rates, and we show that the real interest rate is indeed mildly counter-cyclical or at best acyclical, depending on the empirical series we use. The negative or zero correlation between the real rate and output, and the fact that most models of economic fluctuations cannot replicate it, has been reported by [King and Rebelo \(1999\)](#). We provide alternative measures of the real rate and confirm previous findings.

Finally, we now provide some clarification about the mapping between income shares in the model and in US data. We choose to divide the non-labor's share into a profits' share and a capital share, and in so doing we deviate from much of previous research. Separating payments to capital from payments to shareholders is most consistent with the quantitative framework we present in subsequent sections. In the model we describe below there are three uses of firms' income: payments to labor, payments to capital, and payments to shareholders (profits resulting from monopoly power). We model firms as monopolistic competitors as a convenient mechanism to model firm entry and exit. But this model choice has a clear implication: equity and capital are distinct concepts in our environment. Capital holders get rents because capital is a productive factor of production and equity holders get rents because firms have monopoly power. Therefore the separation between profits and capital payments is natural. In the data, it is not trivial how to separate those two concepts. By taking corporate profits as our empirical measure of pure profits and the remaining components of the aggregate net operating surplus as payments to capital, we make a specific assumption.<sup>6</sup> This assumption is that corporate profits represent monopoly rents

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<sup>6</sup>Proprietors' income is part of the net operating surplus and our preferred measure of labor's share

as producers manufacture differentiated commodities to consumers. Arguably, a fraction of profits measured in national accounting is not measuring pure profits, but rather measuring income from intangible capital or representing miscalculations when imputing interest or rental income of the physical capital corporations own. Unfortunately, we have no good estimates of what that fraction might be. We have taken the view that all measured profits are monopoly rents as opposed to the view that they represent only payments to capital. <sup>7</sup>

## 2 The Model Economy

### 2.1 Environment

Our economy is populated by a large extended household comprised of a continuum of members of total mass equal to  $\bar{N}$  and an infinite mass of firms. The assumption of a large family in which members share unemployment risk is fairly standard, and it avoids dealing with heterogeneity which eliminates tractability.

Members in the household can either be employed or unemployed. Unemployed agents receive an unemployment benefit while they search for jobs with the hope of finding a job opportunity. This opportunity will allow them to enter into a relationship with a firm, to negotiate a contract that stipulates the compensation for their services, and to produce output during the following period. A fraction  $N_t$  of agents works and gets paid the negotiated wage. Members of the household have preferences over a sequence of a composite of goods over time,  $\{C_t\}_{t=0}^{\infty}$ . The per-period utility function is of the relative risk aversion class. The household's (expected) discounted lifetime utility as of time 0 is given by,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} \right], \quad (1)$$

where  $\beta \in (0, 1)$  is the discount factor and  $\sigma > 0$  is the coefficient of relative risk aversion. We assume that each firm produces a differentiated commodity. Having firms as monopolies achieves two goals: first, it is convenient for modeling firm entry and two, it generates profits which compensate firms for paying the sunk cost. At each point in time, there is a subset of goods  $X_t \subseteq X$  available to consumers, and the composite good is made up of commodities

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includes a fraction of proprietors' income as a compensation to labor. Of the remaining components of the net operating surplus, rental income and net interest are the two largest components of payments to capital.

<sup>7</sup>Over the sample period we consider, the average share of profits is 8%, the average share of capital is 29%, and the average share of labor is 63%.

from that subset. The available set is time-varying as not all firms will produce every period. To aggregate over different commodities, we use a [Dixit and Stiglitz \(1977\)](#) aggregator:

$$C_t = \left( \int_{x \in X_t} [c_t(x)]^{\frac{\gamma-1}{\gamma}} dx \right)^{\frac{\gamma}{\gamma-1}}, \quad (2)$$

where  $\gamma > 1$  is the symmetric elasticity of substitution between commodities. If  $p_t(x)$  is the price of product  $x$ , then the level of  $c_t(x)$  chosen to minimize the cost of acquiring  $C_t$  given prices  $\{p_t(x)\}$  for all  $x$  is:

$$c_t(x) = \left( \frac{p_t(x)}{P_t} \right)^{-\gamma} C_t, \quad (3)$$

where  $P_t$  is the cost of acquiring one unit of the composite good, or the price index<sup>8</sup>:

$$P_t = \left( \int_{x \in X_t} [p_t(x)]^{1-\gamma} dx \right)^{\frac{1}{1-\gamma}}.$$

Each firm uses capital and one unit of labor to produce its commodity. The job market in our economy is characterized by the existence of search and matching frictions (see [Rogerson, Shimer, and Wright \(2005\)](#) for a survey of this literature). To hire a worker, a firm must post a vacancy and undertake a recruiting expense of  $\omega$  per vacancy posted. Firms and potential workers match in a labor market, according to a constant-returns-to-scale matching technology  $M(\bar{N} - N, V)$  given by:

$$M(\bar{N} - N, V) = \frac{(\bar{N} - N)V}{((\bar{N} - N)^\xi + V^\xi)^{\frac{1}{\xi}}}. \quad (4)$$

This matching function takes as inputs the total number of unemployed individuals who are searching,  $\bar{N} - N$ , and the total number of vacancies posted by firms,  $V$ . The output is a number of matches  $M$ . Denoting by  $\theta$  the vacancies to unemployment ratio  $\frac{V}{\bar{N} - N}$ , the

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<sup>8</sup> $P$  can be obtained by solving the consumer expenditure minimization problem for constructing one unit of composite good:

$$\begin{aligned} P &= \min_c \int_{x \in X_t} p(x) c(x) dx, \\ \text{s.t. } C &= \left( \int_{x \in X_t} [c(x)]^{\frac{\gamma-1}{\gamma}} dx \right)^{\frac{\gamma}{\gamma-1}} = 1. \end{aligned}$$

probabilities that a vacancy gets filled,  $q_t$ , and that a worker finds a job,  $f_t$  are given by<sup>9</sup>,

$$q_t = \frac{M(\bar{N}-N,V)}{V} = \frac{1}{(1 + \theta_t^\xi)^{\frac{1}{\xi}}}, \quad (5)$$

$$f_t = \frac{M(\bar{N}-N,V)}{N-N} = \frac{\theta_t}{(1 + \theta_t^\xi)^{\frac{1}{\xi}}}. \quad (6)$$

A match between a firm and a worker results in a wage contract that specifies a wage  $w_t(x)$ , paid in exchange of labor services. We assume that firms and workers split the surplus from their relationship according to a Nash bargaining rule. We will explore this rule further after we have established the notation regarding workers' and firms' value functions. The relationship between a firm and a worker can break either because the firm exogenously ends production, which happens with probability  $\tau$ , or for any other reason, which happens at rate  $s$ .

Firms need to pay a sunk cost to begin the goods production process.<sup>10</sup> Opening a firm or starting a new product variety needs  $y^E$  effective units of capital, i.e.  $y^E = Z_t K_t^E$ .<sup>11</sup> We assume the productivity process  $Z_t$  is first-order Markov. Denoting by  $r_t$  the rental rate of capital and noting that one unit of capital produces  $Z_t$  units of the composite good, the sunk cost of entry is  $\frac{r_t y^E}{Z_t}$  or  $r_t K_t^E$  (in units of the composite consumption good). We denote the number of entrants, the number of firms that pay the sunk cost, by  $N_t^E$ .<sup>12</sup>

Let us now describe the technology for producing the differentiated commodity, which, as the reader may recall, involves capital and labor. Denoting the firm's output of the differentiated product  $x$  by  $y_t^c(x)$ , we can formally describe that technology as,

$$y_t^c(x) = Z_t l_t(x)^{1-\alpha} (K_t^C(x))^\alpha, \quad (7)$$

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<sup>9</sup>We depart from the more frequent Cobb-Douglas specification for the matching function to bound the job-finding and vacancy-filling probabilities to be between 0 and 1. This functional form was chosen by [Ramey, den Haan, and Watson \(2000\)](#).

<sup>10</sup>Our approach for modeling firm entry follows [Bilbiie, Ghironi, and Melitz \(2012\)](#).

<sup>11</sup>The assumption that entry costs involve capital only (as opposed to capital and labor) could be relaxed. One may assume a competitive labor market for the entry sector and a frictional one for producing commodities, although it would be difficult to find evidence to justify such an assumption. Alternatively, one could assume a frictional labor market in the entry sector. However, that would involve specifying a different matching function, wage, etc. . . . Two frictional labor markets would certainly complicate things.

<sup>12</sup>[Bilbiie, Ghironi, and Melitz \(2012\)](#) abstract from growth as this specification of sunk costs implies that introducing growth in the productivity process  $Z_t$  leads to the absence of a balanced growth path. For this reason, we abstract from growth as well. To restore balanced growth, one could, in principle, introduce growth in  $y^E$  and set that growth rate to be equal to that of  $Z_t$ .

where  $Z_t$  is the same random productivity process that determines the efficiency of capital when paying for the sunk cost, and  $l_t(x)$  is the amount of labor employed by the firm, which is one if the firm produces and zero otherwise. The firm charges a price equal to  $\rho_t(x)$ ,<sup>13</sup> and its profits are given by  $\pi_t(x) = \rho_t(x)y_t^c(x) - w_t(x) - r_tK_t^C(x)$ .

Finally, the government plays a very limited role in our economy. Its task is solely to tax the household a lump-sum quantity and rebate it to the unemployed in the form of a benefit.

## 2.2 Optimization and Equilibrium

We restrict ourselves to a symmetric equilibrium, in which all goods-producing firms charge equal prices,  $\rho_t(x) = \rho_t$ ; demand one unit of labor, which gets paid the same wage  $w_t(x) = w_t$ ,<sup>14</sup> and produce the same amount of output,  $y_t^c(x) = y_t^c$ . Given the CES structure of the consumption aggregate, the relative price  $\rho_t$  that firms charge is given by<sup>15</sup>  $N_t^{\frac{1}{\gamma-1}}$ , and the per-firm profit is given by,  $\pi_t = \rho_t y_t^C - w_t - r_t K_t^C$ . The relevant state vector for the firm is the quadruplet  $(K_t, N_t, V_t, Z_t)'$  with  $K_t = N_t^E K_t^E + N_t K_t^C$ . To minimize notation, we write down value functions without being specific about their dependence on the state vector.

Households own a diversified portfolio of firms, and as a result, firms discount expected future flows taking into account the household's inter-temporal condition. Consequently, a firm's appropriate discount factor between periods  $t$  and  $t + 1$  is,

$$\Delta_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma}. \quad (8)$$

Let  $Q_t$  denote the capital value of a vacancy and  $J_t$  denote the capital value of a filled job. The following two recursive relationships must be satisfied:

$$Q_t = -\omega + (1 - \tau) E_t \Delta_{t+1} [q_t J_{t+1} + (1 - q_t) Q_{t+1}], \quad (9)$$

$$J_t = \pi_t + (1 - \tau) E_t \Delta_{t+1} [(1 - s) J_{t+1} + s Q_{t+1}]. \quad (10)$$

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<sup>13</sup>The relative price  $\rho_t(x)$  is defined as  $\rho_t(x) = p_t(x)/P_t$ . We want to measure firms' profits in units of the composite consumption good, not in units of any individual commodity. As a result, for all purposes the price charged by firms is the relative price  $\rho_t(x)$  and not the absolute price  $p_t(x)$ .

<sup>14</sup>By our assumption of a large household all workers' threat points are equal. Symmetry across firms implies equality across threat points for firms as well (the value of  $Q_t$ ). As a result, all workers get paid the same wage.

<sup>15</sup>Given that  $p_t(x) = p_t$  and  $\rho_t = \frac{p_t}{P_t} = \frac{p_t}{\left( \int_{x \in X_t} [p_t]^{1-\gamma} dx \right)^{\frac{1}{1-\gamma}}}$ , the implication is that  $\rho_t = \frac{p_t}{p_t \left( \int_{x \in X_t} dx \right)^{\frac{1}{1-\gamma}}}$  and as a result,  $\rho_t = \left( \int_{x \in X_t} dx \right)^{\frac{1}{\gamma-1}} = N_t^{\frac{1}{\gamma-1}}$ , as  $N_t$  is the both the fraction of firms producing as well as the number of workers in the goods-producing sector by our assumption of one job per firm.

Equation (9) states that the value of a vacancy (once the entry decision has been made) is the difference between two objects. The first object is the expected value of entering the labor market and trying to match with a worker. This matching happens with probability  $q_t$ , as long as the firm survives for one period, which happens with probability  $1 - \tau$ . The second object is the vacancy cost  $\omega$ .

The interpretation of equation (10) is analogous: the value of a filled job is the profit flow  $\pi$  plus the expected continuation value of the relationship between the firm and the worker. Conditional on the firm's survival, the relationship ends with probability  $s$  and continues with probability  $1 - s$ .

In equilibrium, the entry of firms occurs until the value of a vacancy is equal to the sunk cost,

$$Q_t = r_t K_t^E. \quad (11)$$

Due to entry costs, vacant jobs have positive value in equilibrium, which in turn leads firms to re-post vacancies following separations. The following two equations give the laws of motion for the stock of employment and vacancies:

$$N_{t+1} = (1 - \tau) [(1 - s)N_t + f_t(\bar{N} - N_t)], \quad (12)$$

$$V_{t+1} = (1 - \tau) [(1 - q_t)V_t + sN_t] + N_t^E. \quad (13)$$

Employment at time  $t + 1$  is the sum of matches  $(1 - s)N_t$  that were not destroyed either by the death of a firm or any other form of separation, and the newly-formed matches  $f_t(\bar{N} - N_t)$  from a previous pool of unemployed people. The total number of vacancies in the economy, given by equation (13), is equal to vacancies that did not get filled in the current period,  $(1 - q_t)V_t$  plus the number of separated matches  $sN_t$ . Of course, we need to include the fraction of firms which continue operating for at least one more period. Finally, we need to add to reach the total, the number of newly created firms  $N_t^E$ , each of which posts a vacancy. Both employment and vacancies are predetermined variables.

The household's problem is relatively straightforward. Given its current period resources, it chooses consumption and investment to maximize the expected discounted value of lifetime utility. In addition to wage income and unemployment benefits, the household gets interest from renting capital as well as a pay-out from its diversified ownership stake in firms. The aggregate dividends firms pay out equal to  $d_t = N_t\pi_t - \omega V_t - Q_t N_t^E$ . Finally, the household pays a lump-sum tax  $T_t$ , which the government uses to finance its unemployment benefits program. Denoting by  $W_t$  the household's value function at time  $t$ , the optimization problem

is expressed as:

$$W_t = \max_{C_t, I_t} \frac{C_t^{1-\sigma}}{1-\sigma} + \beta E_t W_{t+1} \quad (14)$$

subject to

$$C_t + I_t = b(\bar{N} - N_t) + w_t N_t + r_t K_t + d_t - T_t, \quad (15)$$

$$K_{t+1} = (1 - \delta) K_t + I_t. \quad (16)$$

The optimal inter-temporal condition is:

$$\beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} (r_{t+1} + 1 - \delta) \right] = 1. \quad (17)$$

As discussed in the previous section, wages for employed workers are the result of Nash bargaining between each worker/firm pair. The surplus of the match for the household is captured by the change in welfare derived from having a marginal unemployed person who is then hired. This change is given by  $\frac{\partial W_t}{\partial N_t}$ , which in units of the consumption good is  $\frac{\partial W_t}{\partial N_t} C_t^\sigma$ . The surplus for the firm is given by  $J_t - Q_t$ , the difference between the value of a filled job and the value of a vacancy. The Nash bargaining solution when the firm's bargaining parameter is given by  $\phi$  satisfies the following surplus-splitting rule:

$$\frac{J_t - Q_t}{1 - \phi} = \frac{C_t^\sigma \frac{\partial W_t}{\partial N_t}}{\phi}, \quad (18)$$

which yields the following equation for wages:<sup>16</sup>

$$w_t = (1 - \phi)b + \phi(\rho_t y_t^C - r_t K_t^C + \omega) - \phi \left( 1 - \frac{V_t}{\bar{N} - N_t} \right) [\omega + Q_t - (1 - \tau) E_t (\Delta_{t+1} Q_{t+1})]. \quad (19)$$

To better understand the analysis on the dynamics of income shares that follows, let us first define these shares. Total output  $y_t$  can be decomposed in three elements: payments to capital, labor, and equity-holders. As a result we can re-write output as,

$$y_t = r_t K_t + N_t w_t + N_t \pi_t. \quad (17)$$

<sup>16</sup>See the appendix for a derivation of the wage equation.

<sup>17</sup>To be clear about how we reach this expression, recall that profits are defined by  $\pi_t = \rho_t y_t^C - r_t K_t^C - w_t$ . Total output  $y_t$  is defined as the sum of output in the two sectors:  $y_t = r_t K_t^E N_t^E + \rho_t y_t^C N_t$ . Simple substitution yields  $y_t = r_t K_t^E N_t^E + N_t(\pi_t + r_t K_t^C + w_t) = r_t K_t + N_t \pi_t + N_t w_t$ .

Labor's share is then defined as  $\frac{w_t N_t}{y_t}$ , and profits' share is defined as  $\frac{N_t \pi_t}{y_t}$ .

We can now describe a symmetric equilibrium for our economy. It is a sequence of prices  $\rho_t, w_t, r_t$ ; a sequence of aggregate quantities  $K_t, C_t, N_t, V_t, N_t^E, \pi_t$ ; and a sequence of value functions  $Q_t, J_t, W_t$  such that for any time period  $t$ , the following conditions hold:

1. (*Household Optimization*) Given prices  $\rho, w, r$ , the household's optimization results in decision rules for  $C_t$  and  $I_t$  and the value function  $W_t$ .
2. (*Factor Market Clearing*) The interest rate  $r_t$  equates the capital demanded by new entrants  $N_t^E$  and current producers  $N_t$  to that supplied by the household, and the wage  $w$  satisfies the Nash bargaining solution given by equation (19).
3. (*Goods Market Clearing*)  $C_t + I_t + \omega V_t = \rho_t y_t^C N_t$ .
4. (*Firm's Optimization*) Given the demand for a differentiated commodity given by equation (3),  $\rho_t$  is the profit-maximizing price for the monopolist. Aggregate labor demand and vacancies posted by all firms,  $N_t^E, N_t$  and  $V_t$ , satisfy equations (12) and (13), and the vacancy and filled position values satisfy equations (9) and (10).
5. (*Entry Condition*)  $Q_t = r_t K_t^E$ .
6. (*Government*) The government satisfies its budget constraint:  $b(\bar{N} - N_t) = T$ .

## 2.3 Calibration

We calibrate the model to the monthly frequency by assigning values to parameters, so that steady-state moments in the model match those observed in U.S. data. The risk aversion coefficient  $\sigma$  is set to 1.5, well within the range of values typically used in studies of aggregate fluctuations. The discount factor  $\beta$  is set to  $0.99^{\frac{1}{3}}$ , implying a steady-state interest rate equal to 4.1% per annum.

We assume that the productivity process  $Z_t$  follows an AR(1) process with persistence parameter  $\rho_z$  and a zero-mean normally distributed shock with variance  $\sigma_\epsilon^2$ . We set  $\rho_z = 0.964$  and  $\sigma_\epsilon = 0.0052$ , which are consistent with the cyclical persistence and variance in the observed Solow residual.<sup>18</sup> Lacking direct evidence on a reasonable value for the workers'

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<sup>18</sup>In the presence of monopolistic competition, variations in the Solow residual cannot be directly associated with productivity of factors of production. The computation of the Solow residual assumes perfect competition and only then can that association be made. For an extensive discussion, see [Hornstein \(1993\)](#).

bargaining parameter  $\phi$ , we set it equal to 0.5 to make our results comparable to the existing literature (e.g. [Shimer \(2005\)](#)).

We calibrate the exit probability  $\tau$  and the separation rate  $s$  following a procedure similar to that of [Ramey, den Haan, and Watson \(2000\)](#). Let  $\Sigma$  be the total job separation rate caused either by a firm's death or any other cause. The rate at which firms exit the market and do not repost vacancies is  $\tau$ , while  $(1 - \tau)s$  is the rate at which workers separate from firms but where firms re-post vacancies immediately. Hence,  $\Sigma = \tau + (1 - \tau)s$ . The fraction of vacancies that are reposted immediately after separations is then  $\frac{(1-\tau)s}{\Sigma}$ . Denote this quantity by  $\Omega$ . Note also that  $\Sigma N$  gives the total flow out of employment, and as a result,  $\Omega q \Sigma N$  gives the total number of posted vacancies filled. If we subtract the number of posted vacancies that are filled from the total flow out of employment, we get the steady-state mass of jobs that is destroyed permanently:  $\Sigma N - \Sigma N \Omega q = \Sigma N(1 - \Omega q)$ . In a steady state, job destruction must equal job creation. The empirical evidence described by [Shimer \(2005\)](#) sets  $\Sigma$  equal to 0.1 at the quarterly frequency, which implies  $1 - (1 - 0.1)^{\frac{1}{3}} = 0.035$  at the monthly frequency. Therefore,

$$\Sigma = (1 - \tau)s + \tau = 0.035. \quad (20)$$

[Davis, Haltiwanger, and Schuh \(1998\)](#) report that the job-creation-to-employment ratio in the manufacturing sector is 0.052 quarterly, which implies a value of 0.018 at the monthly frequency. [Blanchard and Diamond \(1989\)](#) argue that vacancy postings have an average of 3 weeks, which implies that the vacancy filling rate is  $1 - (1 - 1/3)^4 = 0.802$  per month. Given this value of  $q = 0.802$  per month,

$$\frac{\text{Job Creation}}{\text{Employment}} = \frac{\Sigma N(1 - \Omega q)}{N} = 0.018. \quad (21)$$

From equations (20) and (21) we can solve for  $s = 0.021$  and  $\tau = 0.014$ .

Consistent with estimates reported by [Basu and Fernald \(1997\)](#), we set  $\gamma = 11$ , which implies a markup of 10 percent. Changing the total mass of workers  $\bar{N}$  only amounts to changing the levels, i.e., the scale of output and the mass of employment, etc. But the unit-free ratios (e.g., unemployment rate, v-u ratio, and consumption-output ratio etc.), are unaffected. Therefore, a choice of  $\bar{N}$  does not affect any of the second moments and the impulse responses. We choose  $\bar{N} > 1$  so that the monopolist's price is larger than the resulting price if markets are competitive, given by  $\lim_{\gamma \rightarrow \infty} N_t^{\frac{1}{\gamma-1}} = 1$ .

We are left with six parameters to calibrate:  $(b, y^E, \delta, \omega, \xi, \alpha)$ . To do so, we choose six additional moments that the model needs to match in its steady state. Based on his own

Table 1: Summary of Parameterization

Parameter	Value	Target/Source
$\sigma$	1.500	Prev. work
$\phi$	0.500	Prev. work
$\rho_z$	0.964	NIPA
$\sigma_z$	0.005	NIPA
$\tau$	0.018	$\frac{Job\ Creation}{Employment} = 0.018$
$s$	0.017	$\Sigma = 0.035$
$\gamma$	11	10% markup
$\beta$	$(0.99)^{\frac{1}{3}}$	$r=4.1\%$
$\delta$	0.002	$K/Y = 36.67$
$b$	0.381	$b/w = 0.426$
$y^E$	2,862	$\omega V/Y = 0.015$
$\omega$	0.525	$f = 0.45$
$\xi$	1.630	$\theta = 0.561$

calculations, [Shimer \(2005\)](#) documents that the monthly job finding rate is 0.45. Note that the steady state value of market tightness can be written as  $\theta = \frac{f}{q} = 0.56$ . We choose to match the aggregate capital to aggregate output ratio, and we set it to a value of 36.67, which implies a value of 3.05 at the annual frequency (the average for our sample). We set the total recruiting costs as a fraction of output,  $\omega V/Y$  to be 1.5%. A controversial choice is that of the value of the unemployment benefit  $b$ . Much of the literature argues that the value of non-work activities is far below what workers actually produce on the job. However, calibrations such as [Hagedorn and Manovskii \(2008\)](#) claim success in terms of the cyclical properties of the model when the outside option for workers is very close to their productivity. Under the interpretation of  $b$  as purely monetary unemployment benefits, we set  $b$  so that the steady-state replacement ratio  $b/w$  is 0.426, as in [Shimer \(2005\)](#) and [Gertler and Trigari \(2009\)](#). Finally, we want to match an additional moment in the steady-state value for labor's share, which is 0.625 for the sample under consideration. In conclusion, to assign values to the vector of parameters  $(b, y^E, \delta, \omega, \xi, \alpha)$ , we choose the following six moments:  $f = 0.45$ ,  $\theta = 0.56$ ,  $(\omega V)/Y = 0.15$ ,  $K/Y = 36$ ,  $b/w = 0.426$ , and  $wN/Y$ .

We summarize our parameterization in [Table 1](#).

## 3 Results

### 3.1 The Cyclical Behavior of Labor’s and Corporate Profits Shares

Having assigned parameter values to the model, we solve it, simulate it, and judge its implications against U.S. data. Our solution technique is standard: we approximate the true solution by a first order expansion around the model’s deterministic steady state. Since the calibration is done at the monthly frequency, we transform the model’s output by aggregating its “monthly” data into “quarterly” data by taking three-month averages. We transform the model’s output the same way we transform actual data: we de-trend them by taking logs and applying an HP filter.<sup>19</sup>

#### 3.1.1 Cross-Correlations with Output

How well does the model of costly firm entry match the cross-correlations shown in Figures 1 and 2 compared to standard models where entry is free? Let’s begin with labor’s share.<sup>20</sup> Figure 3 displays the empirical cross-correlations, the same values as Figure 1 represented by the dotted and dash-dotted lines, along with the cross-correlations from the entry model (labeled “Model” in the figure and represented by the circled line). Labor’s share in the model matches the patterns observed in the data remarkably well. The contemporaneous correlation is weak with a value of -0.13 and within the error bounds provided for the empirical correlations. The correlations with one lead and one lag quantitatively match their empirical counterparts, and only correlations at higher leads and lags are somewhat stronger than those found in the data. Most importantly, the model gets the lagging pattern of labor’s share right: after an increase in output, labor’s share shows the largest increase four quarters later without a large contemporaneous effect. Figure 4 shows the analog to Figure 3 for the profits’ share rather than for labor’s share. The figure shows that the model with costly entry matches well the correlations at several leads and lags of the profits: share and output. In fact, all correlations are within the error bounds constructed for the empirical point estimates. Before we explain the pattern of correlations in the model with costly entry, let’s compare it to a benchmark model: the model with free entry. Readers can think of this

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<sup>19</sup>The HP smoothing parameter we use is 1,600, a standard choice when using quarterly data.

<sup>20</sup>All results presented in the paper have CES preferences, see equation (2). This specification features constant markups. To introduce time-varying markups we changed the utility function to be of the translog type, see Feenstra (2003). The results are similar to the baseline CES case and they are available upon request.

model as a version of the one constructed by [Andolfatto \(1996\)](#) and [Merz \(1995\)](#).<sup>21</sup> Figure 5 adds to Figure 3 the patterns of correlations between output and labor’s share computed from the model where entry is free. The figure shows how the labor’s share is strongly countercyclical. Although it may not be apparent from a quick glance at the graph, the value of the contemporaneous correlations is -0.93, as opposed to -0.13 in the costly entry model and -0.11 in the data. These differences are large.<sup>22</sup> The performance of the free entry model regarding the profits share is better, as one can see in Figure 6, but considerably worse than the costly entry model. In the free entry model, the strong cyclicality in the profits share, which is consistent with the data, comes at the expense of a strong cyclicality in the labor’s share, which is not. De-linking the cyclical dynamics of the two shares, in the sense of generating weak correlations between labor’s share and output and relatively stronger correlations between the profits’ share and output, is something our model is able to achieve.

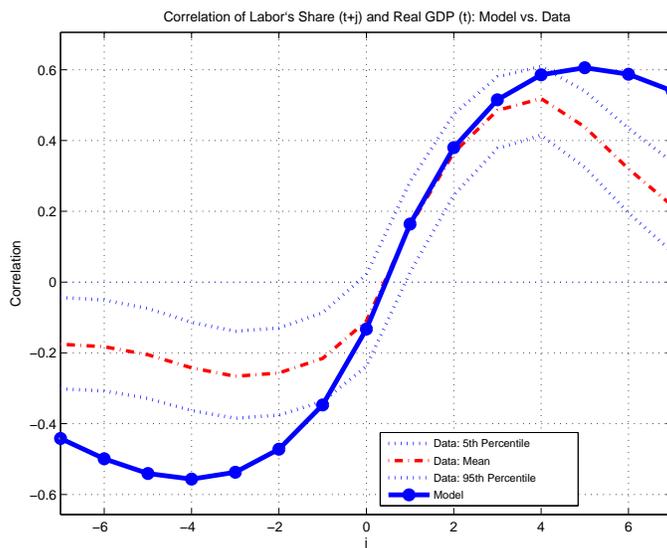


Figure 3: Correlations between Real Gross Domestic Product and (led and lagged) Labor’s Share of Real GDP - US Data 1951-2007 (dashed-dotted lines) and Costly Entry model (circled line)

<sup>21</sup>We describe with more detail the structure and the calibration of the free-entry model in the Appendix.

<sup>22</sup>Both [Andolfatto \(1996\)](#) and [Merz \(1995\)](#) report significantly smaller correlations (-0.625 and -0.768, respectively), but still much larger in absolute value than those found in the data. There is an important distinction between the structure of our free entry model and [Andolfatto’s](#) or [Merz’s](#). We abstract from search intensity and a labor-leisure choice. Given that our first calibration is similar to theirs, we hypothesize that the quantitative difference in the correlation of labor’s share with output is caused by that element missing from our framework.

The strong counter-cyclicality of labor’s share generated in the free entry model is caused by the relatively larger response of output to a rise in productivity. Matching frictions prevents employment from adjusting immediately to a productivity shock, a feature of all models presented in this paper. In the free entry model, both wages and output respond rapidly to a change in productivity, but output responds relatively stronger. As a result, labor’s share falls sharply (relative to its steady state value) but as employment rises in subsequent periods, labor’s share rises as well. This movement explains the strong negative contemporaneous correlation and the mildly positive correlation of output with the value of labor’s share three or four quarters later.

To understand the dynamics in the costly-entry model, remember that output  $y_t$  is equal to,

$$\rho_t y_t^C N_t + r_t K_t^E N_t^E = \rho_t y_t^C N_t + Q_t N_t^E.$$

In other words, total output is the sum of income in two “sectors”: the commodity producing sector,  $\rho_t y_t^C N_t$ , and the “start-up” sector,  $r_t K_t^E N_t^E$ . The joint dynamics of both sectors determine the dynamics of total output. In the case of a positive productivity shock, an immediate response is a drop in  $K^E$ , the “per-start-up” amount of capital, as  $y^E$  is constant. The number of entrants  $N^E$  rises, as the present value of profits is now higher. The remaining key variable determining the behavior of output in the “start-up” sector is therefore the interest rate,  $r_t$ . The equilibrium interest rate is determined by the relative demand and supply of capital in the two sectors. Total capital,  $K_t = N_t^E K_t^E + N_t K_t^C$ , is a predetermined variable but the economy can reallocate it intra-temporally between the two sectors. The technology of the goods producing sector being Cobb-Douglas forces interest rates to rise in the face of a positive technology shock. This result is standard in models of economic fluctuations with Cobb-Douglas technology and the culprit for the strong procyclicality of real interest rates in the real business cycle literature. What happens in the “start-up” sector? Because  $K_t^E$  falls when  $Z_t$  rises, the demand for capital by any given entrant is lower, forcing interest rates to drop. In summary, the behavior of interest rates in the face of an increase in productivity is the result of two counteracting forces. On the one hand, technology in the goods producing sector pulls interest rates upward when productivity rises, but on the other hand, it lowers the amount of capital an entrant needs, lowering the demand for capital and pulling rates downward. Using the calibration described previously, interest rates are countercyclical. This drop in interest rates is responsible for the more muted response of output (relative to that of wages) in the costly entry model. In turn, this drop also dampens

the negative response of labor’s share to an increase in productivity.

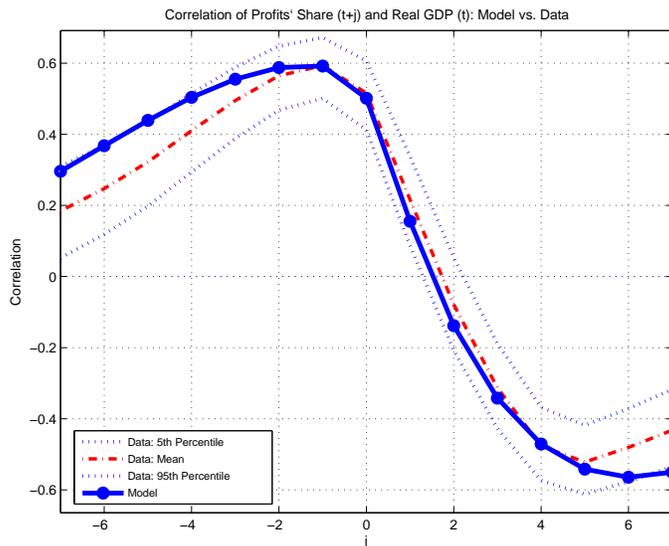


Figure 4: Correlations between Real Gross Domestic Product and (led and lagged) Profits’ Share of Real GDP - US Data 1951-2007 (dashed-dotted lines) and costly entry model (circled line).

The evolution of interest rates helps explain the more muted response of output in the costly entry model. However, this factor is only part of the story when it comes to explaining the different dynamics of labor’s share in the two models. The existence of entry costs may, depending on parameter values, make the response of employment (and wages) persist over time. This persistence is due to resources in the start-up sector competing with those in the goods-producing sector. As a result, entrants may find it optimal to delay their entrance so more capital can be used for producing goods when productivity is high. This delayed response raises wages and employment for several quarters, increasing the numerator in the expression for labor’s share, explaining the lagging behavior, that is, the high positive correlation between output and the value of labor’s share four quarters later. The performance of the costly entry model regarding the profits’ share can be analyzed using the same intuition. Profits rise when productivity, and hence output, rises; they are pro-cyclical. The share of profits in total output is also pro-cyclical, as employment does not react immediately to changes in productivity. The persistence of employment and wages, which increase for several periods after a rise in productivity, cause the negative correlation of profits shares several quarters in the future, with contemporaneous output.

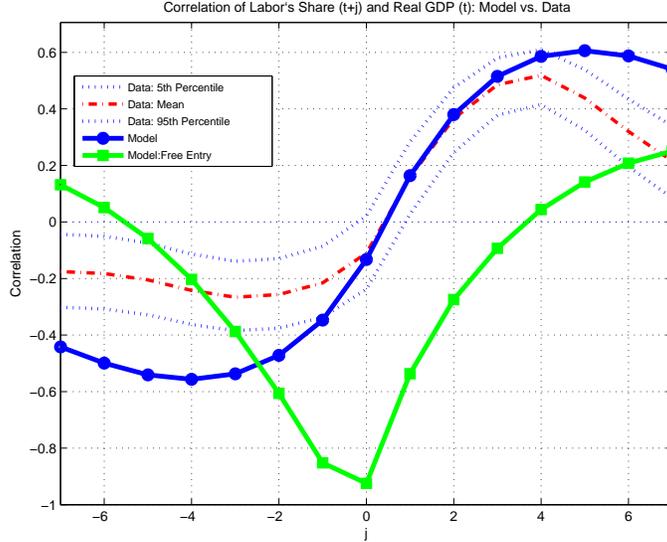


Figure 5: Correlations between Real Gross Domestic Product and (led and lagged) Labor's Share of Real GDP - US Data 1951-2007 (dashed-dotted lines), costly entry model (circled line) the free entry model (HM) (squared line), and the free entry model (line with plus signs).

So far, we have seen that the joint dynamics of firm entry, the asset value of a vacant position, and the interest rates are important for understanding the dynamics of income shares. In truth, distinguishing between the dynamics of the value of a vacancy and interest rates in the costly entry models is unnecessary. Recall that the value of a vacancy  $Q_t$  is equal to  $r_t K_t^E$ . In both costly entry models,  $K_t^E$  displays exactly the same dynamics, again because  $y^E$  is a constant and  $Z_t$  is exogenous. So the behavior of  $Q$  is essentially driven by the behavior of interest rates,  $r$ . But let us return to understanding the dynamics of income shares by showing through a different channel that it is indeed the joint dynamics of entry and real interest rates that are crucial. They are crucial both for the weak low contemporaneous correlation between output and the labor's share and the lagging pattern of the cross-correlations.

This different channel is running an experiment that involves making firm entry less attractive by lowering the efficiency of the matching technology. This efficiency is represented by  $\xi$ , which we set now to a value of 0.38. (It was 1.630 before). This lower efficiency has two effects. First, it lowers the steady-state value of entrants. As matching becomes more difficult, the probability of matching to a worker decreases. This decrease lowers the prospects of making any profits, leading to a lower level of entrants in equilibrium. A lower level of

entrants in equilibrium implies that the demand for capital is determined mainly in the goods producing sector, leading to a procyclical real interest rate. The second effect is the lower persistence of employment: as each firm has a lower probability of matching with a worker employment does not rise as much following a rise in productivity. These two effects make the response of output closer to that of the free entry model, response that implies a larger contemporaneous drop in labor's share. The less persistent response of employment and wage prevents the numerator of labor's share to rise much in subsequent periods, reducing the correlation between output and labor's share after four quarters. Figure 7 shows the disappearance of the lagging behavior and the appearance of a strong countercyclicality of labor's share. That figure shows that the contemporaneous correlation between the labor's share and output is close to -1. Profits shares also display similar dynamics to the free entry model (see Figure 8). To re-iterate, these differences arise because of changing dynamics in firm entry, interest rates, and the asset value of a vacant position. They do not arise because of differences in the level of sunk costs  $y^E$ , which has remained at the value calibrated in the previous section throughout the exercise of lowering  $\xi$ .

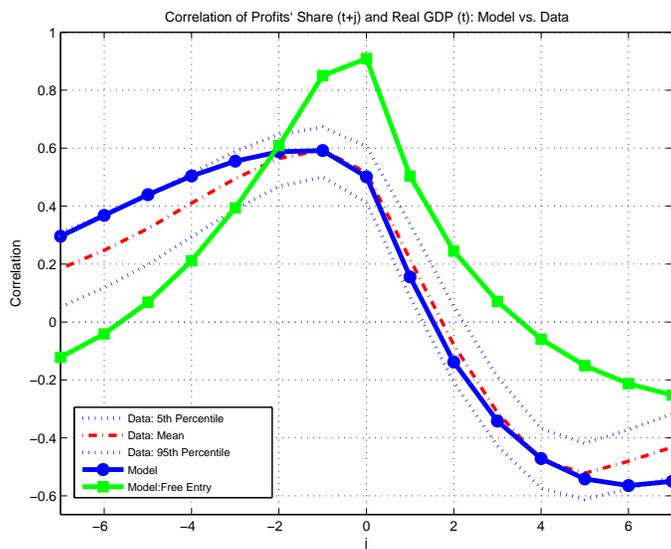


Figure 6: Correlations between Real Gross Domestic Product and (led and lagged) Profits' Share of Real GDP - US Data 1951-2007 (dashed-dotted lines), costly entry model (circled line), the free entry model (HM) (squared line), and the free entry model (line with plus signs).

The previous figures, and the intuition behind them, make it clear that the behavior of entrants, the behavior of the value of a vacancy, and the dynamics of interest rates are

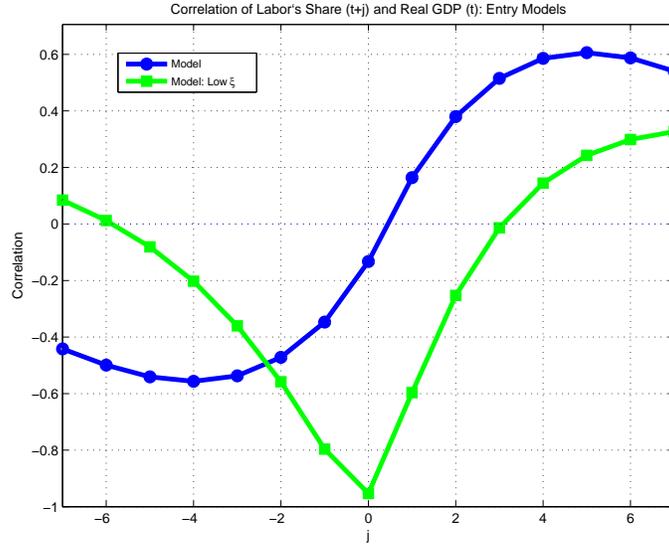


Figure 7: Correlations between Real Gross Domestic Product (output) and (led and lagged) Labor's Share of output - costly entry model (circled line) and Low  $\xi$  model (squared line).

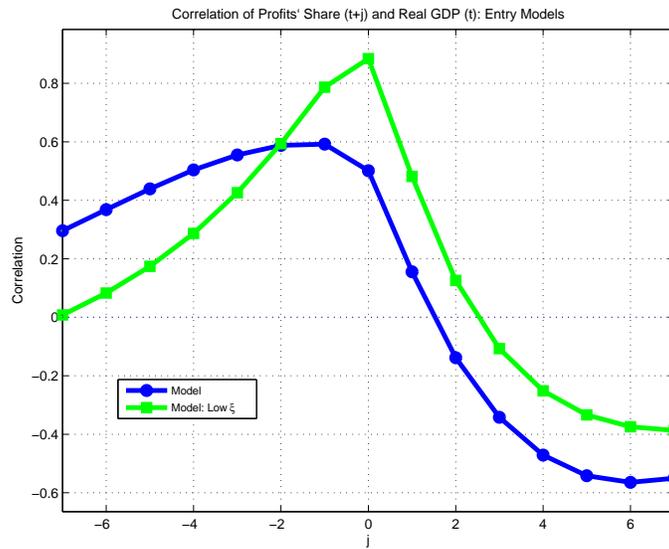


Figure 8: Correlations between Real Gross Domestic Product (output) and (led and lagged) Profits' Share of output - costly entry model (circled line) and Low  $\xi$  model (squared line).

important for understanding the dynamics of income shares. To further validate our model we need more evidence. To that end, we show that the dynamics of real interest rates in the data are consistent with the costly entry model at the expense of the other three (the

costly entry model with a low value of  $\xi$  and the two versions of the free entry model). To get an empirical counterpart to interest rates in the theoretical models, we first obtain quarterly measures of nominal yields from corporate bonds (Aaa-rated). We restrict the sample as the same as that used to compute correlations of income shares: 1951:Q2-2007:Q1. To transform those nominal yields into real yields, we subtract the inflation rate for that quarter measured as the quarter-over-quarter growth rate the Consumer Price Index (CPI) (annualized, because yields are annualized as well).<sup>23</sup> The first row of Table 2 shows the correlation at the quarterly frequency of real interest rates and output in the data and in the three model economies.<sup>24</sup> The correlation between the real interest rate and output in the data is -0.323,<sup>25</sup> and that negative correlation is reported by King and Rebelo (1999) as a stylized fact that macroeconomic models have a difficult time replicating. However, one ought to be worried when equating interest rates from bonds to the real interest rate in our model, which represents the cost of renting capital. Gomme, Ravikumar, and Rupert (2011) compute the correlation between alternative measures of the return to capital that are perhaps better proxies to what we call the real interest rate than yields on corporate bonds. They find that (de-trended) synthetic returns to business capital constructed from NIPA data have a correlation with output of 0.16. While there are obvious difficulties in measuring the return to capital empirically, it is clear that all these correlations are very weak and likely negative. The free entry model and the costly entry model with low  $\xi$  feature very pro-cyclical interest rates with correlations close to 1 (0.972, and 0.989 respectively). On the other hand, the Costly Entry model features a countercyclical real interest rate: its correlation with output is -0.31.<sup>26</sup>

Given the correlations of interest rates and output in the two costly entry models, it is not surprising that the correlation between the value of a vacancy is negative in the costly entry model and positive in the costly entry model with low  $\xi$ . Given the tight link in the

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<sup>23</sup>We take current inflation as a reasonable forecast of inflation in the next three months. In the short-run, this “random-walk” forecast works remarkably well (see Stock and Watson (1999b)).

<sup>24</sup>We de-trend real interest rates, both in the data and in the model economies, by computing the percentage deviation relative to steady state.

<sup>25</sup>Stock and Watson (1999a), using expected inflation calculated using a VAR and the yield on T-bills (a very short-term interest rate), report a correlation of -0.35.

<sup>26</sup>Dotsey, Lantz, and Scholl (2003) report that the negative correlation is sensitive to the definition of the price index used to calculate the inflation rate. In particular correlations computed using the Personal Consumption Expenditures Price Index (PCE), instead of the Consumer Price Index (CPI), can be positive or zero for particular inflation-forecasting procedures, but always small in absolute value. With our “random-walk” forecast, the correlation between the real interest rate, computed using the growth rate in the PCE price index as the inflation measure, and output, is -0.30.

Table 2: Correlations between  $y_t$  and  $r_t$ ,  $Q_t$  and  $N_t^E$ : Data vs. Models

	US Data	Costly Entry	Costly Entry (low $\xi$ )	Free Entry
$Corr(y, r)$	-0.323	-0.310	0.989	0.972
$Corr(y, Q)$	N/A	-0.684	0.934	N/A
$Corr(y, N^E)$	0.510	0.982	0.988	N/A

model between  $Q_t$  and  $r_t$ , even though we lack empirical measures of the value of a vacancy, the model shows that the real interest rate is a good proxy. This proxy strengthens the hypothesis that the costly entry model is a good representation of the data.

Finally, to the best of our knowledge, no good measures of firm entry exist. We have taken one used by [Bilbiie, Ghironi, and Melitz \(2012\)](#), in which they report a correlation of 0.510 with output (see their Figure 2). However, even if good measures of entry existed, this variable does not allow one to distinguish among the two costly entry models. The reason is that firm entry is procyclical and similar in magnitude in both models.

### 3.1.2 The Response of Interest Rates

The mechanism behind the countercyclical behavior of the interest rate can be better understood using Figure 9. The top plot displays the response of the real rate  $r_t$  to a positive shock to  $Z_t$ . The middle and bottom plots display the responses for the total amount of capital employed in the entry sector ( $N_t^E K_t^E$ ) and in the commodities-producing sector ( $N_t k_t^C$ ) respectively. The positive innovation to productivity makes capital in both sectors more productive. As entry costs are constant, any given entrant requires a smaller quantity of  $K^E$ . But since profits are expected to rise, entry becomes more attractive and the quantity of entrants rises. Even so, the total amount of capital devoted to finance entry costs ( $N^E K^E$ ) falls on impact. As (total) capital and employment are fixed, the commodities-producing sector needs to absorb a larger amount of capital, precipitating the drop in the interest rate. Thereafter the drop in productivity  $Z$  more than compensates the pressure upwards on interest rates coming from a higher stock of employment (or operating firms). Hence interest rates keep falling.<sup>27</sup>

<sup>27</sup>Once the level of employment peaks, interest rates start converging to the steady-state from below.

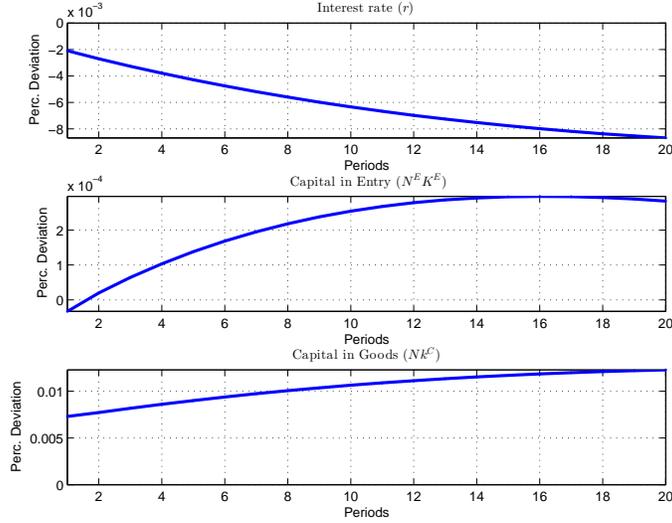


Figure 9: Response to a productivity shock of the real interest rate (top plot), total capital employed in the entry sector (middle plot), total capital employed to produce commodities (lower plot).

### 3.1.3 Volatility of Income Shares Relative to Output

As an end to the section we examine the performance of the four model economies with respect to the volatility of income shares relative to output. Table 3 reports the relative volatilities of labor's share (first row) and profits' share (second row) relative to that of output in US data and in the four models. In US data, labor's share is roughly half as volatile as output and profits' share is approximately five times as volatile as output. Quantitatively none of the models shown get these relative volatilities right, but it's clear that costly entry improves those statistics. The free entry model cannot even match qualitatively the volatilities of income shares: the volatility of labor's share is higher than the volatility of profits' share. Both are relatively smooth, particularly the profits' share which is about 36 times more volatile in the data than in the model. The costly entry model, on the other hand, is at least consistent with a more volatile profits' share: its standard deviation is about three-quarters the value in the data. Unfortunately, this high volatility also implies a high volatility for labor's share which is almost twice as volatile as in the data. The reason for the large volatility in profits shares in the costly entry model is the slow response of vacancies and employment to changes in  $Z_t$  (including the slow response of entrants). Profits rise with an increase in productivity as production rises without a corresponding rise in wages. As

Table 3: Volatility of Income Shares Relative to Output: Data vs. Models

	US Data	Costly Entry	Free Entry
$\sigma_{LS}/\sigma_{GDP}$	0.453	1.890	0.169
$\sigma_{PS}/\sigma_{GDP}$	5.190	3.786	0.107

wages rise eventually, profits fall rapidly as labor’s share rises.

## 4 Final Remarks

We have constructed a quantitative model of the macroeconomy that is consistent with most income shares’ time series facts. The novel aspect of our environment relative to models with frictional labor markets is to assume costly entry by firms. This assumption introduces cyclical dynamics in the asset value of a vacant position, a value which in equilibrium has to equal the expenditures undertaken by firms to enter production markets. For the model to account for income shares’ dynamics and to propagate and amplify productivity shocks, the asset value of a vacancy has to be negatively correlated with output over the business cycle. In our framework, interest rates have to be negatively correlated. This negative correlation of real interest rates and output has proven difficult to obtain in production economies.

Although the framework can account for many time series facts regarding labor markets and income shares times, work remains to be done. For instance, labor’s share seems to have a high volatility at low frequencies but a low volatility at high frequencies. This translates to a large and persistent response of that share to changes in productivity. The cyclical component of labor’s share is smoother than output. These two facts are difficult to reconcile with the type of model we have presented and call for further research to account for low-frequency movements in income shares of a different nature than the high frequency movements observed between expansions and recessions.

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