The Impact of Uncertainty on Two-Tiered

Labor Markets*

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Abstract

This paper studies the impact of higher aggregate uncertainty on labor markets where jobs with strong employment protection coexist with temporary contracts. We develop a search and matching model where firms and workers choose the type of contract. The model allows for endogenous separations and job-to-job transitions. We show that in the data, times of heightened uncertainty correlate with a higher share of temporary workers, a lower wage inequality between permanent and temporary jobs, and a slight increase in unemployment. Our model is consistent with these facts. Higher uncertainty increases the option value of temporary contracts relative to permanent contracts, changing the composition of the labor force but with minor changes in overall employment levels.

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1 Introduction

The goal of this paper is to understand the behavior of dual labor markets – a segmentation of the workforce by job security and duration – when aggregate uncertainty rises. Temporay contracts are a flexible tool for employers when facing uncertainty, but concerns about their effect for workers' earnings growth and inequality are commonplace. To understand this tradeoff, this paper answers the following questions: what determines the aggregate fraction of temporary and permanent jobs? How does the composition of the labor market change after a persistent rise in aggregate uncertainty? What are the implications for aggregate productivity and inequality? The answers to these questions are essential for understanding labor markets and to tailor redistributive policies to markets with strong employment protection. In answering these questions, we make theoretical, empirical, and quantitative contributions.

Our theoretical contribution is to present an environment in which the fractions of temporary and permanent jobs are endogenous. The model highlights matchquality as the variable determining the initial choice of contract. By match-quality, we refer to the component of a worker's productivity that remains fixed as long as the firm and the worker do not separate. That component is revealed at the time the firm and worker meet. Firms offer workers with low match-quality a temporary contract, which can be terminated at no cost after one period and features a relatively low wage. If it is not terminated, the firm agrees to promote the worker and upgrade the contract into permanent. A permanent worker enjoys a higher wage and is relatively protected by a firing cost. However, firms find it optimal to offer high-quality matches a permanent contract. Because workers can search on the job, the firm is better off as it avoids losing a good match, given the lower turnover rate of permanent positions.

Our set-up is tractable enough to allow us to characterize three cut-off rules. These rules summarize the hiring and firing decisions of firms. First, we derive a cut-off point in the distribution of match-specific shocks above which the firm offers a permanent contract. Below that cut-off point, the firm offers a temporary contract. There is also a cut-off point in the distribution of the time-varying component of productivity below which the relationship between a temporary worker and a firm ends and above which it continues. Finally, we show the existence of a cut-off point also in the distribution of the time-varying component of productivity. Below that point, the relationship between a permanent worker and a firm ends; above that point, it continues.

Our empirical contribution, and to link the model with the data, is to first document facts on the Canadian dual labor market drawing from different sources. We choose to study Canada for several reasons. It has a sizable share of temporary workers (13%) and high quality labor market data. Also, Canada has one of the least stringent labor market legislations in the OECD, with relatively low protection for permanent relative to temporary workers. For that reason, it is a better example of the sorting between temporary and permanent workers based on productivity as opposed to other countries with more stringent employment protection legislation. Moreover, its economy experienced an increase in aggregate uncertainty after the 2009 recession. Using the Labor Force Survey (LFS), we find that permanent workers earn roughly 33% more per hour than temporary workers. We use the Workplace and Employee Survey (WES), a matched employer-employee dataset, to estimate the time-varying productivity component of workers' output. We also use this panel to calculate different employees' firing and hiring rates (including promotions). Finally, an important ingredient of our model is on-the-job search. From the Survey of Labour and Income Dynamics (SLID), we estimate that the rate of job-to-job transitions for temporary workers is more than double that of permanent workers.

Our main quantitative question is what happens on a two-tiered labor market after an increase in aggregate uncertainty. We measure uncertainty by the dispersion of the time-varying component of productivity, a common measure at the disaggregate level (see Bloom, 2014). We focus on the labor force composition by contract types and the earnings distribution. In the data, the dispersion increased by 7.1% post-crisis. Simultaneously, the share of temporary workers out of total employment increased from 12.6% to 13.2%. The average wage premium that permanent workers enjoy dropped by 3%. Finally, the average unemployment rate slightly went up from 8.4% to 8.5%. We calibrate the economy with data up to 2008. We perturb that economy with a one-time aggregate shock to the dispersion of the time-varying productivity distribution. Only with that change (no other changes in parameter values), the model can quantitatively match the increase in the fraction of temporary workers, the drop in the wage ratio between permanent jobs and temporary jobs, and the slight rise in unemployment after the 2008 financial crisis.

Why does the model generate these changes after a rise in productivity dispersion? The mechanism responsible for the changes in labor markets is novel, and it is related to overall preference for temporary contracts that a rise in uncertainty brings. Hiring a low match-quality worker has more upside; the higher variance of the productivity distribution makes higher outcomes more likely. This increases the probability of a future permanent contract. It is true that very low outcomes are also more likely but in that case the firm can part with the worker for free. Permanent contracts, on the other hand, are now less attractive. Under a permanent contract, a worker that has a low productivity value is expensive to fire. These low productivity outcomes are now more likely, due to the rise in uncertainty. To summarize, a rise in uncertainty induces more temporary jobs offered relative to permanent jobs.

This asymmetric impact on contract types affects labor market turnover. Permanent workers are more likely to lose jobs, while temporary workers are less likely to be fired. Because the fraction of permanent workers accounts for more than 75% of the total labor force, the outflow from permanent jobs to unemployment dominates inflows from retention of temporary workers. Hence, unemployment slightly increases. Because of the higher relative value of temporary contracts, In response to a rise in uncertainty, firms are reluctant to offer permanent contracts. Some high-quality matches, which were offered permanent contracts in a low uncertainty environment, become temporary workers. As a result, the average match-quality for temporary jobs increases, which leads to a higher average wage for temporary workers.

1.1 Related Literature

This paper contributes to the literature on two-tiered labor markets by explaining the choice between permanent and temporary jobs. Cahuc et al. (2016) introduce different expected durations of production opportunities into the search and matching model. In their model, the termination of a temporary job is costly before the expiration of the contract. Thus, it provides incentives for firms to hire permanent workers to exploit production opportunities with long durations. In our environment, the trade-off the firm faces when choosing between temporary and permanent contracts is different: a permanent contract is a way to mitigate the risk of losing a high-quality match. Their paper focuses on the duration of temporary jobs and the impact of employment protection. In contrast, we investigate the consequence of changes in aggregate uncertainty. Berton and Garibaldi (2012) propose a directed search model in which firms and workers self-select into permanent and temporary contracts. The key trade-off is between the low filling rate of temporary jobs and the high firing cost of permanent jobs. The model predicts that temporary workers have a shorter unemployment spell than permanent workers. Alonso-Borrego et al. (2004) and Caggese and Cuñat (2008) assume a firm hires both permanent and temporary workers. The coexistence of both contracts is due to some degrees of imperfect substitution: either permanent jobs generate higher productivity or have different hiring costs than temporary ones.

Our study is related to the growing literature on the interaction between uncertainty and labor market dynamics. Recent studies have found that an increase in uncertainty impacts the labor market negatively. Leduc and Liu (2016), Pries (2016), and Schaal (2017) show that an uncertainty shock can cause higher unemployment. While available work focuses on labor market dynamics over the business cycle, ours investigates a large and persistent one-time change in aggregate uncertainty due to a large event such as the Great Recession. This simpler approach enables us to understand better the role of uncertainty on different labor contracts and their performance, about which the extant literature is silent. We identify a new channel through which uncertainty can affect the labor market, namely through the *composition* of temporary and permanent employment. At the same time this approach precludes a comparison between the quantitative effects of uncertainty shown below to other works using models of aggregate fluctuations.

The rest of this paper is organized as follows. Section 2 describes the model environment. Section 3 characterizes the equilibrium and shows the cut-off rules for hiring and firing decisions. Section 4 describes the data used, documents labor market facts, and calibrates the model. It also includes the analysis of the impact of an increase in uncertainty on labor market outcomes. Section 5 concludes. All the proofs and additional derivations can be found in the appendix.

2 Economic Environment

We present a version of the Mortensen and Pissarides (1994) model with on-thejob search and two types of employment contracts: temporary and permanent. Our labor market is populated by a unit mass of *ex-ante* identical workers who are endowed with one unit of time each period. Workers can either be employed or unemployed as a result of being fired and hired by firms. The mass of firms is potentially infinite. Workers search for jobs and firms post vacancies with the hope of matching to searching workers. The number of meetings between employers and workers is given by a matching technology that we specify below in detail. Employment contracts between firms and workers can be of two types: permanent or temporary. A permanent contract has no predetermined length, but firms and workers renegotiate wages every period. Separating from this kind of contract is costly. Under a permanent contract, if a firm and a worker separate, firms pay a firing tax *f* that we assume is wasted. On the other hand, a temporary contract has a predetermined length of one period; at the end of that period separating from the match comes at no cost to the firm. In the case the match continues the temporary contract is upgraded to a permanent one. This upgrade costs the firm a small fee *c*.¹ While on the job, external offers arrive at a rate λ^T and λ^P respectively for temporary and permanent workers. We assume throughout that $\lambda^T \ge \lambda^P$ because it is consistent with the empirical evidence.² Unemployed workers receive benefits *b* for as long as they are unemployed. The government finances the unemployment insurance program by levying lump-sum taxes τ on both workers and unemployed agents.

The production technology does not depend on the contract signed. If a firm hires worker *i*, the match yields $z_i + y_{i,t}$ units of output in period *t*. The random variable *z* represents match-quality and hence, as long as the match lasts *z* is a constant. This component of a worker's productivity is drawn from a distribution G(z) and it is revealed at the time a firm and a worker meet. The time-varying component $y_{i,t}$ is drawn every period from a distribution F(y) and it is responsible for endogenous separations. We assume both shocks to be independent across agents and time. The supports of both distributions are given by $[y_{min}, y_{max}]$ and $[z_{min}, z_{max}]$ and we assume throughout that $y_{min} < y_{max} - c - f$.

A matching technology $B(v, N^S)$ determines the number of pairwise meetings between workers searching for a job (N^S) and employers (represented by the number of vacancies posted v). This technology displays constant returns to scale and implies a job-finding probability $\alpha^w(\theta)$, and a vacancy-filling probability $\alpha^f(\theta)$, which are both functions of the level of market tightness $\theta \equiv \frac{v}{N^S}$. We assume that

¹Given that match-quality z is bounded, the introduction of a promotion cost prevents a firm from offering a temporary contract to a worker with the highest possible match-quality, only to promote the worker with certainty after one period. By doing so, the firm can save on the firing cost for at least one period without any downside. To be consistent with the hiring rules derived later, we assume that promotion is costly so that firms have an incentive to offer permanent contracts to the most suitable workers in the first place.

²One can endogenize the search decision by adding a search cost. There exists an equilibrium which only temporary workers will search on-the-job, while permanent workers won't. The reason is that the "return" of searching on the job is lower for those high match-quality workers. These are precisely the workers hired under permanent contracts, and as a result there is an interval of search costs such that permanent workers do not search but temporary workers do.

the market tightness lies in a bounded interval $[\theta_{min}, \theta_{max}]$.³ Every time a firm decides to post a vacancy, it must pay a cost *k* per vacancy posted. We denote by *Q* the value of an unfilled vacancy.

We divide a period into three different stages or sub-periods. The first stage is the search and matching stage. Unemployed workers search for jobs, while employed workers possibly receive on-the-job offers. Firms post vacancies which get matched to workers. Upon matching, match-quality z is realized and conditional on its value firms and workers decide whether to sign a contract and of which type.

The second sub-period is a separation stage where the time-varying productivity y is realized in this stage. Conditional on the value of y, the firm decides to separate with the worker or not, or promote the worker to a permanent contract if the worker was hired as a temporary worker in the last period.⁴

The third stage is the production stage. During this stage matches deliver z + y units of output. Firms obtain profits after paying workers wages that are contingent on the type of contract signed. Unemployed workers receive *b* in the form of unemployment benefits.

A worker's output makes having a job filled valuable for the firm. Nevertheless, the value of a job filled with a permanent worker differs from that of a job filled with a temporary worker. Denote by $J^P(z, y)$, $J^T(z, y)$, $J^P_0(z, y)$, and $J^R(z, y)$, the values of a job filled respectively with a permanent worker (with productiv-

$$heta_{min} = \xi^{\frac{1}{\phi}},$$

 $heta_{max} = \xi^{-\frac{1}{1-\phi}}.$

³A standard Cobb-Douglas matching function $B(v, N^S) = \xi v^{1-\phi} (N^S)^{\phi}$ satisfies this assumption. The implied matching probabilities are $\alpha^w(\theta) = \xi \theta^{1-\phi}$ and $\alpha^f(\theta) = \xi \theta^{-\phi}$. Since the matching probability is between 0 and 1, the implied market tightness must be in a bounded interval where

⁴In other words, the newly formed matches in the first stage are also subject to separation.

ities *z* and *y*), a temporary worker, a new permanent hire, and a just-promoted permanent worker.⁵ As is standard, the value of a filled job can be defined by the sum of a flow profit plus a continuation value. The continuation value takes into account the probability of losing the worker at either the search and matching or the separation stages.⁶

A proposition below demonstrates that match-quality z determines the choice between a temporary or a permanent contract. It is convenient to define by,

$$A \equiv \left\{ z \in \left[z_{min}, z_{max} \right] | E_y \left[\max \left(J_0^P \left(y, z \right), Q \right) \right] \ge E_y \left[\max \left(J^T \left(y, z \right), Q \right) \right] \right\}$$

the set of realizations of z for which the firm prefers to offer a permanent contract. Also for convenience, let \mathbb{I}_A denote an indicator function defined as,

$$\mathbb{I}_A = egin{cases} 1 & z \in A, \ 0 & z \notin A. \end{cases}$$

At the beginning of any given period workers are either employed or unemployed. Employed workers can be of two types: employed under a permanent contract or employed under a temporary contract. Nevertheless, the value of being a permanent worker (or having a filled permanent job) depends on whether the worker is a new hire, a promotion, or a continuation of a permanent contract.⁷ Denote by $U, V^P(z, y), V^T(z, y), V^R(z, y), V_0^P(z, y)$ respectively the value of starting

⁵The values of a job filled with a permanent worker who has been just promoted or of a job filled with a worker who has been recently hired, are different from a job filled with an existing permanent match. Because there is no firing cost to pay in case the worker does not get promoted or when the firm is deciding to sign a permanent contract with a new matched-worker, the wages the firm pays are different in each of these three cases. As a result, the value a filled job is also different in each of the three cases.

⁶In the Appendix, we provide more detail about the equations determining the values for workers of being unemployed or employed, as well as the values of the different filled-jobs.

⁷The reason for that difference is the same as that explaining the difference in the values of job when it is filled with a new permanent hire, a new promotion, or an existing permanent worker.

the period: (a) unemployed, (b) employed under a permanent contract with productivities *z* and *y*, (c) employed under a temporary contract, (d) having just been promoted to a permanent worker, and (d) finding a new permanent job. Each of these values is the sum of an income flow plus a continuation value. The income flow for the unemployed is *b*, the level of benefits, and for the employed is the wage.⁸ Finally, let $V^N(z)$ be the value of a new job conditional on a match-specific productivity value *z*. Using the definitions provided above, this value is given by,

$$V^{N}(z) \equiv \mathbb{I}_{A}E_{y}\left[\max\left(V_{0}^{P}(y,z),U\right)\right] + (1 - \mathbb{I}_{A})E_{y}\left[\max\left(V^{T}(y,z),U\right)\right].$$

The value of a new job is simply the value of choosing between a permanent contract and unemployment if z is in A. Alternatively, if z is not in A, the value of a new job is the choice between unemployment and a temporary contract.

Suppose a worker is employed under a permanent contract with match-quality \hat{z} . Let $B^{P}(\hat{z})$ be the set of match-quality values such that quitting his current job is preferable for the worker. Formally,

(1)
$$B^{P}(\hat{z}) \equiv \left\{ z \in [z_{min}, z_{max}] \middle| V^{N}(z) \ge E_{y} \left[\max \left(V^{P}(y, \hat{z}), U \right) \right] \right\}.$$

The left-hand side of the inequality in (1) is the value of accepting a new job offer. To determine whether the move is optimal, the worker compares that value to the value of remaining with the current employer (the right-hand side). Similarly, the switching policy of a temporary worker involves defining the set of match-

⁸In the appendix we provide detailed expressions determining the values of being employed or unemployed, as well as for each of the values of a filled job.

specific productivities for which the move is optimal. This set is given by,

(2)
$$B^{T}(\hat{z}) \equiv \left\{ z \in [z_{min}, z_{max}] \middle| V^{N}(z) \ge E_{y} \left[\max \left(V^{R}(y, \hat{z}), U \right) \right] \right\}.$$

Given these two sets, the probability that the worker switches jobs conditional on a successful match for $j \in \{P, T\}$ is

$$ho^{j}\left(\hat{z}
ight)\equiv\int_{z\in B^{j}\left(\hat{z}
ight)}dG\left(z
ight).$$

In other words, the worker switches jobs if he receives an on-the-job offer with match-quality \hat{z} in $B^{j}(\hat{z})$. Given a distribution G(z), the probability of that occurring is $\rho^{j}(\hat{z})$.

Since the work of Burdett and Mortensen (1998) it is well known that workers may use on-the-job search to obtain a higher wage in the same type of job, i.e., wage dispersion arises even for homogeneous jobs. It is also well known that models of random matching introducing on-the-job search are frequently intractable. To highlight the main mechanism and to keep the analysis tractable, we follow Pissarides (1994) and many others in adopting two simplifying assumptions regarding wage determination. The first assumption is that wages are set according to a linear surplus-splitting rule that entitles workers to a fraction $\phi \in (0, 1)$ of flow rents. The second assumption is that the wage can be revised continuously at no cost, so that long-term contracts are ruled out. In the case of an on-the-job offer, this assumption implies that the new employer immediately renegotiates the wage once the worker breaks the relationship with the previous employer. Thus, the current employment status does not affect the wage offer received from out-

side, even if the worker starts negotiations with a new employer before resigning from the current job. The first assumption implies that the worker and the firm act efficiently, since they receive a constant fraction of the match surplus. In other words, a match is operated if and only if its total return is higher than that of unemployment. These assumptions lead to a wage-setting rule that looks identical to the typical Nash bargaining solution in models without on-the-job search.

3 Equilibrium

3.1 Hiring and Firing Rules

We assume that wages are set so that firms and workers linearly split the total surplus of the match. The fraction that accrues to a worker is governed by a parameter ϕ . Each contract implies a different surplus, and hence a different negotiated wage. Generally, the gain for a firm from entering (or continuing) a match is the difference between having a filled job and having an unfilled vacancy. If a permanent match is dissolved, the firm also pays *f*, the firing cost. The gain for a worker of entering or continuing a match is the difference between being employed in that match and being unemployed. For each type of contract, adding up the gains for a firm and for a worker results in the following expressions for the total surpluses:

(3)

$$S^{P}(y,z) = J^{P}(y,z) - (Q - f) + V^{P}(y,z) - U,$$

$$S^{P}_{0}(y,z) = J^{P}_{0}(y,z) - Q + V^{P}_{0}(y,z) - U,$$

$$S^{R}(y,z) = J^{R}(y,z) - Q + V^{R}(y,z) - U,$$

$$S^{T}(y,z) = J^{T}(y,z) - Q + V^{T}(y,z) - U.$$

As a result of the bargaining assumption, surpluses satisfy the following splitting rules:

4)

$$S^{P}(y,z) = \frac{J^{P}(y,z) - Q + f}{1 - \phi} = \frac{V^{P}(y,z) - U}{\phi},$$

$$S^{P}_{0}(y,z) = \frac{J^{P}_{0}(y,z) - Q}{1 - \phi} = \frac{V^{P}_{0}(y,z) - U}{\phi},$$

$$S^{R}(y,z) = \frac{J^{R}(y,z) - Q}{1 - \phi} = \frac{V^{R}(y,z) - U}{\phi},$$

$$S^{T}(y,z) = \frac{J^{T}(y,z) - Q}{1 - \phi} = \frac{V^{T}(y,z) - U}{\phi}.$$

(4

Given these, the total expected surplus of a new job is

$$S^{N}(z) \equiv \mathbb{I}_{A}E_{y}\left[\max\left(S_{0}^{P}(y,z),0\right)\right] + (1 - \mathbb{I}_{A})E_{y}\left[\max\left(S^{T}(y,z),0\right)\right].$$

Substituting for the values for being unemployed and employed under different contracts in (4), the surpluses can be rewritten as

$$S^{P}(y,z) = y + z - \tau + \beta \left(1 - \lambda^{P} \alpha^{w}(\theta) \rho^{P}(z)\right) E_{y} \left[\max\left(S^{P}(y,z),0\right)\right] \\ + \phi \beta \lambda^{P} \alpha^{w}(\theta) \int_{B^{P}(z)} S^{N}(\tilde{z}) dG(\tilde{z}) \\ - (1 - \beta) U + \left[1 - \beta \left(1 - \lambda^{P} \alpha^{w}(\theta) \rho^{P}(z)\right)\right] f,$$
(5)

$$S_{0}^{P}(y,z) = y + z - \tau + \beta \left(1 - \lambda^{P} \alpha^{w}(\theta) \rho^{P}(z)\right) E_{y} \left[\max\left(S^{P}(y,z),0\right)\right] \\ + \phi \beta \lambda^{P} \alpha^{w}(\theta) \int_{B^{P}(z)} S^{N}(\tilde{z}) dG(\tilde{z}) \\ - (1 - \beta) U - \beta \left(1 - \lambda^{P} \alpha^{w}(\theta) \rho^{P}(z)\right) f,$$
(6)

$$S^{R}(y,z) = y + z - c - \tau + \beta \left(1 - \lambda^{P} \alpha^{w}(\theta) \rho^{P}(z)\right) E_{y} \left[\max\left(S^{P}(y,z),0\right)\right] \\ + \phi \beta \lambda^{P} \alpha^{w}(\theta) \int_{B^{P}(z)} S^{N}(\tilde{z}) dG(\tilde{z}) \\ - (1 - \beta) U - \beta \left(1 - \lambda^{P} \alpha^{w}(\theta) \rho^{P}(z)\right) f,$$
(7)

and

$$S^{T}(y,z) = y + z - \tau + \beta \left(1 - \lambda^{T} \alpha^{w}(\theta) \rho^{T}(z)\right) E_{y} \left[\max\left(S^{R}(y,z),0\right)\right] \\ + \phi \beta \lambda^{T} \alpha^{w}(\theta) \int_{B^{T}(z)} S^{N}(\tilde{z}) dG(\tilde{z}) \\ - (1 - \beta) U,$$
(8)

The following proposition establishes the existence of thresholds, or cut-off values, for the time-varying productivity component *y*. These thresholds determine lower-bounds for the continuation of a match.

Proposition 1. Unique cutoffs $y^{P}(z)$, $y_{0}^{P}(z)$, $y^{R}(z)$ and $y^{T}(z)$ exist, if the following conditions hold:

(9)
$$y_{max} + z_{min} \geq b + \frac{\phi}{1-\phi} \theta_{max} k + \beta f + c,$$

(10) $b - \left[1 - \beta \left(1 - \lambda^{P} \alpha^{w} \left(\theta_{max}\right)\right)\right] f \geq y_{min} + z_{max} + \beta \int_{y_{min}}^{y_{max}} \left(1 - F\left(y\right)\right) dy$.

In other words, a value of *y* larger than $y^{P}(z)$ implies a continuation of an existing permanent contract. If *y* is larger than $y^{R}(z)$, a temporary worker becomes a permanent worker. Finally, if *y* is larger than $y^{T}(z)$, a recent temporary hire continues the relationship with the firm and may be promoted to a permanent hire in the subsequent period. Conditioning these thresholds on a certain match-quality *z* is intuitive.

The assumption that *y* is independent over time does not imply that firms and

workers wish to continue the match irrespective of the drawn value of *y*. The firm would rather repost an unfilled vacancy than employ a worker whose productivity is known to be low. This logic applies to any worker-firm pair irrespective of the type of contract. However, the cut-off point is itself contract-specific because the cost of undoing a permanent match differs from that of undoing a temporary match.

The following two lemmas are useful for proving Proposition 2 below.

Lemma 1. If $G'(z) < 1/(\beta \lambda^P f)$, then $S^j(y,z)$ is strictly increasing in z, and the separation rules $y^j(z)$ is strictly decreasing in z.

Lemma 2. If $\phi \to 1$ and $\lambda^P \to 0$, $dy^T/dz > dy_0^P/dz$.

The next proposition establishes the existence of a threshold in the distribution of match-quality z that determines the contract choice upon a meeting between a firm and a worker. Values of z larger than the threshold result in permanent contracts; otherwise the firm and the worker enter a temporary contract.

Proposition 2. There is a unique cutoff \overline{z} such that for any $z \ge \overline{z}$, the firm will offer permanent contract, while for $z < \overline{z}$ the firm will offer temporary contract, if the f and c are sufficiently large.

According to Propositions 1 and 2, the equilibrium can be characterized by simple cutoff rules. The separation and promotion decision, $y_0^P(z)$ and $y^T(z)$ are downward sloping with z. This is intuitive: for low match-quality, the firm requires a high draw of y to compensate in order to keep or promote the worker. As z goes up, the need for this compensation will go down. From the proof of Proposition 2, the intersection of y_0^P and y^T pins down the cutoff \bar{z} in which the type of contract will be offered. Figure 1 illustrates the equilibrium cutoffs. High

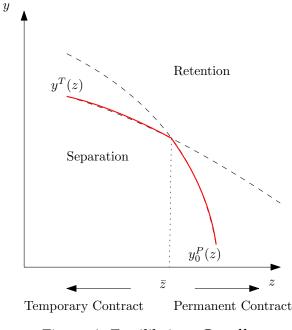


Figure 1: Equilibrium Cutoffs

match-quality values lead to permanent contracts. The firm chooses to offer a higher wage risking having to pay f if the match dissolves. It is optimal to do so because since the expected productivity is high (because match-quality is high) the chances of separating from the match are rather low. Offering a temporary contract to a good match is less preferable, because although the firm pays a low wage, it risks losing the high-match-quality worker. Frictions in the labor market increase the cost of losing a well matched worker.

3.2 Laws of Motion

In our environment, equilibrium prices and allocations depend on the stationary distribution of workers across temporary employment, permanent employment, and unemployment. To calculate this distribution it is convenient to begin calculating transition probabilities for each worker of starting in any of those three states and moving into any other. From these transition probabilities it is easy to derive the associated laws of motion for agents in each of the three states. And from those, calculating the stationary distribution is straightforward.

Let π_{ij} be the probability that a worker in state *i* transits to state *j*. Using these transition probabilities we can write the law of motion for the mass of workers under permanent contracts as,

(11)
$$n_{+1}^{P} = \pi^{UP} u + \pi^{PP} n^{P} + \pi^{TP} n^{T}.$$

Let us derive now the value for the three transition probabilities in that expression. An unemployed individual becomes part of the pool of permanent workers if: (a) they meet a firm during the search and matching stage (which happens with probability $(1 - G(\bar{z}))\alpha^w(\theta)$), and (b) they are not separated in the separation stage, which happens with probability

$$\frac{1}{1-G(\bar{z})}\int_{\bar{z}}^{z_{max}}\left(1-F\left(y_{0}^{P}\left(z\right)\right)\right)dG\left(z\right).$$

Let ρ^{PT} and ρ^{PP} be the probabilities that a currently permanent worker transits to a temporary contract or a permanent contract, respectively, conditional on matching with a firm. A permanent worker may switch to another permanent job with probability $\lambda^{P} \alpha^{w}(\theta) \rho^{PP}$.⁹ It is also possible that he gets an offer, decides not to switch because the draw of *z* is lower than z^{P} , therefore remaining in the permanent workers pool only by drawing a high value of productivity *y*. This event happens with probability,

$$\lambda^{P} \alpha^{w} \left(\theta\right) \int_{z_{min}}^{z_{max}} G\left(z^{P}\left(z\right)\right) \left(1 - F\left(y^{P}\left(z\right)\right)\right) dG\left(z\right).$$

⁹We provide definitions of the four conditional switching probabilities ρ^{PP} , ρ^{PT} , ρ^{TP} , and ρ^{TT} in the appendix.

Finally, the worker may not get a new job offer this period, and he remains in the pool of permanent workers by drawing a high value of *y*. This third scenario occurs with probability $(1 - \lambda^{P} \alpha^{w}(\theta)) \int_{z_{min}}^{z_{max}} (1 - F(y^{P}(z))) dG(z)$

A transition from temporary to permanent employment can occur through three channels. First, directly from temporary employment into permanent by receiving an acceptable on-the-job offer. The probability of such a transition is $\lambda^T \alpha^w(\theta) \rho^{TP}$. Second, by receiving an unacceptable job offer but still passing the promotion threshold which happens with probability,

$$\lambda^{T} \alpha^{w}\left(\theta\right) \frac{1}{G\left(\bar{z}\right)} \int_{z_{min}}^{\bar{z}} G\left(z^{T}\left(z\right)\right) \left(1 - F\left(y^{R}\left(z\right)\right)\right) dG\left(z\right).$$

Third, as in the case of a current permanent worker, a worker under a temporary contract may not get an on-the-job offer. Drawing a productivity value higher than the corresponding threshold will promote the worker to a permanent contract.

Summing up all possible cases for each of the three states, the transition probabilities into a permanent contract are given by:

$$\begin{split} \pi^{UP} &= \alpha^{w}\left(\theta\right) \int_{\bar{z}}^{z_{max}} \left(1 - F\left(y_{0}^{P}\left(z\right)\right)\right) dG\left(z\right), \\ \pi^{PP} &= \lambda^{P} \alpha^{w}\left(\theta\right) \rho^{PP} + \lambda^{P} \alpha^{w}\left(\theta\right) \int_{z_{min}}^{z_{max}} G\left(z^{P}\left(z\right)\right) \left(1 - F\left(y^{P}\left(z\right)\right)\right) dG\left(z\right) \\ &+ \left(1 - \lambda^{P} \alpha^{w}\left(\theta\right)\right) \int_{z_{min}}^{z_{max}} \left(1 - F\left(y^{P}\left(z\right)\right)\right) dG\left(z\right), \\ \pi^{TP} &= \lambda^{T} \alpha^{w}\left(\theta\right) \rho^{TP} + \lambda^{T} \alpha^{w}\left(\theta\right) \frac{1}{G\left(\bar{z}\right)} \int_{z_{min}}^{\bar{z}} G\left(z^{T}\left(z\right)\right) \left(1 - F\left(y^{R}\left(z\right)\right)\right) dG\left(z\right) \\ &+ \left(1 - \lambda^{T} \alpha^{w}\left(\theta\right)\right) \frac{1}{G\left(\bar{z}\right)} \int_{z_{min}}^{\bar{z}} \left(1 - F\left(y^{R}\left(z\right)\right)\right) dG\left(z\right). \end{split}$$

Analogously, the mass of temporary workers evolves according to,

(12)
$$n_{+1}^T = \pi^{UT} u + \pi^{PT} n^P + \pi^{TT} n^T.$$

where the transition probabilities are defined as

$$\begin{aligned} \pi^{UT} &= \alpha^{w}\left(\theta\right) \int_{z_{min}}^{\bar{z}} \left(1 - F\left(y^{T}\left(z\right)\right)\right) dG\left(z\right), \\ \pi^{PT} &= \lambda^{P} \alpha^{w}\left(\theta\right) \rho^{PT}, \\ \pi^{TT} &= \lambda^{T} \alpha^{w}\left(\theta\right) \rho^{TT}. \end{aligned}$$

The interpretation of the conditional job-switching probabilities ρ^{TT} and ρ^{PT} is the same as in the case of permanent workers.¹⁰. Moreover, the cases that lead to transitions out of each of three states into temporary employment follow closely the case of permanent workers. Nonetheless, transitions in this case are somewhat simpler to derive because a permanently employed worker can only transit into temporary employment through a direct on-the-job offer.

Finally, the mass of unemployed workers evolves according to

(13)
$$u_{+1} = \pi^{UU} u + \pi^{PU} n^P + \pi^{TU} n^T.$$

An unemployed worker can remain unemployed because he fails to meet a firm and hence get a job offer. However, he can also remain unemployed by accepting a job offer but failing to draw a sufficiently high productivity value in the separation stage. Taken together, these two cases imply a value for π^{UU} equal to,

$$1 - \alpha^{w}\left(\theta\right) + \alpha^{w}\left(\theta\right) \left[\int_{\bar{z}}^{z_{max}} F\left(y_{0}^{p}\left(z\right)\right) dG\left(z\right) + \int_{z_{min}}^{\bar{z}} F\left(y^{T}\left(z\right)\right) dG\left(z\right)\right].$$

¹⁰We also provide formal definitions of ρ^{PT} and ρ^{TT} in the Appendix.

Three different shocks may generate a transition from a permanent job into unemployment.

First, a permanent worker may accept an on-the-job offer but fails to advance to the production stage because he draws too low a value of y. He enters the unemployment pool as a result. The joint probability of those two events is,

$$\pi_{1}^{PU} = \lambda^{P} \alpha^{w}\left(\theta\right) \int_{z_{min}}^{z_{max}} \int_{z \in [z^{P}(\hat{z}), z_{max}] \cap [z_{min}, \bar{z}]} F\left(y^{T}\left(z\right)\right) dG\left(z\right) dG\left(\hat{z}\right)$$

Second, he may reject an on-the-job offer only to unsuccessful at the separation stage with his current employer. This happens with probability,

$$\pi_{2}^{PU} = \lambda^{P} \alpha^{w}\left(\theta\right) \int_{z_{min}}^{z_{max}} G\left(z^{P}\left(z\right)\right) F\left(y^{P}\left(z\right)\right) dG\left(z\right).$$

Third, the worker does not receive an on-the-job offer, but separates with the current employer with probability

$$\pi_{3}^{PU} = \left(1 - \lambda^{P} \alpha^{w}\left(\theta\right)\right) \int_{z_{min}}^{z_{max}} F\left(y^{P}\left(z\right)\right) dG\left(z\right).$$

Adding up the three cases yields the overall probability of being a permanent worker but ending up unemployed equal to $\pi^{PU} = \pi_1^{PU} + \pi_2^{PU} + \pi_3^{PU}$.

The probability of transiting from temporary employment into unemployed is given by,

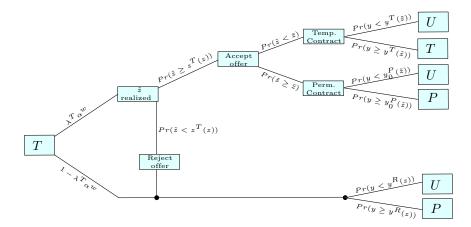


Figure 2: Transitions from a temporary contract and their associated probabilities.

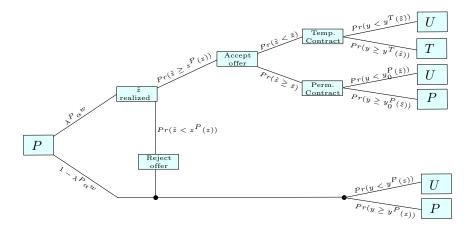


Figure 3: Transitions from a permanent contract and their associated probabilities.

$$\begin{aligned} \pi^{TU} &= \lambda^{T} \alpha^{w} \left(\theta \right) \frac{1}{G\left(\bar{z}\right)} \int_{z_{min}}^{\bar{z}} \int_{z \in [z^{T}(\hat{z}), z_{max}] \cap [z_{min}, \bar{z}]} F\left(y^{T}\left(z\right)\right) dG\left(z\right) dG\left(\bar{z}\right) \\ &+ \lambda^{T} \alpha^{w} \left(\theta\right) \frac{1}{G\left(\bar{z}\right)} \int_{z_{min}}^{\bar{z}} G\left(z^{T}\left(z\right)\right) F\left(y^{R}\left(z\right)\right) dG\left(z\right) \\ &+ \left(1 - \lambda^{T} \alpha^{w}\left(\theta\right)\right) \frac{1}{G\left(\bar{z}\right)} \int_{z_{min}}^{\bar{z}} F\left(y^{R}\left(z\right)\right) dG. \end{aligned}$$

The events that end up in separation into unemployment for a temporary worker are virtually the same as for a permanent worker.

To summarize all possible transitions from either type of employment, Figures 2 and 3 describe all events that lead from temporary and permanent employment, respectively, to other three states.

3.3 Wages

Wages are contingent on the type of contract because the surplus itself is contingent on the type of contract. By our assumption of a linear-splitting rule of the total surplus and using the definitions for the value of unemployment and the value of a job filled with a permanent worker, we can solve for the wage, given by, ¹¹

$$w^{P}(y,z) = \phi \left\{ y + z + \left[1 - \beta \left(1 - \lambda^{P} \alpha^{w} \rho^{P}(z) \right) \right] f \right\} + (1 - \phi) b$$

$$(14) \qquad \qquad + \phi \left(1 - \phi \right) \left(\beta \alpha^{w} \int_{z_{min}}^{z_{max}} S^{N}(\tilde{z}) dG(\tilde{z}) - \beta \lambda^{P} \alpha^{w} \int_{\tilde{z} \in B^{P}(z)} S^{N}(\tilde{z}) dG(\tilde{z}) \right)$$

Similarly, the wages for a new permanent worker, either through new hire or promotion, are

(15)
$$w_0^P(y,z) = w^P(y,z) - \phi f,$$

(16)
$$w^{R}(y,z) = w^{P}(y,z) - \phi f - \phi c.$$

There is a wedge between the wage of an existing permanent match relative to that of a new hire or a promotion. The wedge is the result of the firm not having to pay the firing cost when it decides not to promote a temporary worker, or decides not to hire one. In the former case the firm does need to pay a promotion cost c.¹² The wage for a temporary worker is found by using the linear-splitting rule and the values of temporary employment and unemployment.

¹¹In calculating the wages, we have set the value of a vacancy Q to its equilibrium value of 0. The reason it is zero is that firms are free to enter the market, pay the fee k, and post a vacancy. There can be no rents made from vacancy creation in equilibrium, otherwise new entrants would lower the vacancy-filling rate until k equaled the expected return from a filled job.

¹²As we show in Proposition 2, this cost can be arbitrarily small depending on the specification of shocks. In the quantitative exercise, we set c to 1% of f implying the wage difference between new permanent workers and new promoted workers of 0.5%. In the Appendix, we show how our results are affected if promotion cost is different.

$$w^{T}(y,z) = \phi(y+z) + (1-\phi) b$$
(17)
$$+\phi(1-\phi) \left(\beta \alpha^{w} \int_{z_{min}}^{z_{max}} S^{N}(\tilde{z}) dG(\tilde{z}) - \beta \lambda^{T} \alpha^{w} \int_{\tilde{z} \in B^{T}(z)} S^{N}(\tilde{z}) dG(\tilde{z})\right).$$

In general, if λ^{P} is sufficiently small, then $w_{0}^{P} > w^{T}$.

4 Quantitative Implications

4.1 Data

To quantitatively explore the model, we draw data from multiple sources. As we reported earlier, we use the LFS to obtain the share of temporary employment and wage differentials between temporary and permanent workers. We supplement the LFS with two other data sources, for data moments of job relocation and jobto-job transition.

For rates of job reallocation, we use the Workplace and Employee Survey (WES), a matched employer-employee longitudinal dataset collected by Statistics Canada. Establishments in the WES report their workforce sizes, as well as the total numbers of permanent employees and temporary employees. This information allows us to calculate the establishment-level share of temporary workers, and rates of job creation and job destruction for both types of employment. The share of temporary employment in the WES is largely similar to that obtained from the LFS.

As is typical, we measure turnover by comparing job creation and job destruction rates. If we denote by $EMP_{t,i}$ the total level of employment at time *t* at establishment *i*, the creation and destruction rates between periods *t* and *t* + 1 are calculated as:

(18)
$$Creation = \sum_{i} \frac{Emp_{t+1,i} - Emp_{t,i}}{0.5(Emp_{t+1} + Emp_{t})}$$

if $Emp_{t+1,i} - Emp_{t,i} > 0$ and 0 otherwise. And,

(19)
$$Destruction = \sum_{i} \frac{|Emp_{t+1,i} - Emp_{t,i}|}{0.5(Emp_{t+1} + Emp_{t})}$$

if $Emp_{t+1,i} - Emp_{t,i} < 0$ and 0 otherwise.

Given the emphasis of our work on a labor market segmented by temporary and permanent workers, we use the previous expressions to provide measures of job destruction and creation by the type of contract held. However, we measure creation and destruction of temporary (or permanent) workers relative to the average *total* employment level. In other words, we measure the change in the stock of workers by contract type relative to the stock of total employment. These rates are given on the first two lines of Table 1. The job destruction rates are 6.4% for permanent workers and 6.2% for temporary workers. The creation rates are 8.1% and 5.3%. As the fraction of temporary workers is only 14% of the workforce, these rates point to a much higher degree of turnover for temporary workers.

Notice that the sum of the destruction rates for temporary and permanent workers is not equal to the destruction rate for all workers. The same can be said for the creation rate. The reason is that establishments can change the number of temporary and permanent workers without altering the stock of all workers. If we restrict the sample to those establishments that increase or decrease the stock of both permanent and temporary workers, the rates for all workers are the sum of the rates of the two types of workers. These measures are reported in Table 1 un-

Conventional Definition							
	All Workers	Permanent	Temporary				
Job Creation	10.2	8.1	5.3				
Job Destruction	9.2	6.4	6.2				
Alternative Definition							
	All Workers	Permanent	Temporary				
Job Creation	8.2	5.1	3.1				
Job Destruction	7.1	4.1	3.0				

Table 1: Job Creation and Job Destruction (%)

Note: The table reports measures of job creation and job destruction computed using the WES following definitions (18) and (19). The Conventional Definition considers all establishments. The Alternative Definition considers only establishments that change the total size of their workforce (see main text for details).

der the "Alternative Definition" label. Turnover decreases under this alternative definition, with creation and destruction rates for all workers that are 2% lower than using the conventional definition. The total job creation rate is 8.2% and the job destruction rate is 7.1%.

Wages are reported in the WES, which we cannot use to calculate the average wage differential. This is because only a sample of employees are surveyed for each establishment and the sampled employees are not necessarily representative.

The second additional data source is the Survey of Labour and Income Dynamics (SLID). This is a panel of households similar to the American PSID that provides earnings, employment status, and several other labor market characteristics. From the SLID we compute two statistics employed in the calibration, namely, the job-to-job transitions of permanent and temporary workers. In Canada, these two rates are 0.097 and 0.256, respectively.¹³

¹³One exception is the job-finding probability which we take from Zhang (2008).

4.2 Calibration

Given that the model period is a year, we fixed the annual discount rate to be 0.96. We set c to a small value equal to 1% of f. The matching function is of the form:

$$B(v, N^s) = \frac{vN^s}{\left(v^{\xi} + (N^s)^{\xi}\right)^{\frac{1}{\xi}}},$$

which implies the job-finding and job-filling rates as: ¹⁴

$$\alpha^{w}\left(\theta\right) = \frac{\theta}{\left(1 + \theta^{\xi}\right)\frac{1}{\xi}},$$

and

$$\alpha^{f}\left(\theta\right) = rac{1}{\left(1+ heta^{\xi}
ight)rac{1}{\xi}}.$$

The distribution function of *y* is logistic, and *z* is uniform. In the model the overall scale of the economy in indeterminate and shifts in the mean of *y* plus *z* have no impact. Consequently, we normalize the mean of *y* plus *z* to one, i.e., E(y) + E(z) = 1. There are a total of 10 parameters to which we need to assign a value: ¹⁵

$$\left\{f,k,b,\phi,\xi,\sigma_z,\mu_y,\sigma_y,\lambda^P,\lambda^T\right\}.$$

Some of the parameters have more direct links to empirical moments. For example, when picking f, we target the average firing costs relative to the wage of a permanent worker f/w^p in the data.¹⁶ When calibrating b we match the average

¹⁴In our calibration exercise, we restrict the job-finding rate to fall in [0, 0.999] so that $\theta_{max} < +\infty$.

¹⁵Note that in our calibration, we do not restrict that all parameter values have to satisfy the sufficient conditions listed in Propositions 1 and 2. In stead, we check the decision rules whether are consistent with the implications of the propositions after obtaining a solution. As a result, we verify the monotonicity and uniqueness of the decision rules, and thus ensure the existence of an equilibrium.

¹⁶The fixed firing cost is consistent with employment protection law in Canada. The OECD Employment Protection Legislation (EPL) Database update 2019, documents that various lengths

unemployment insurance replacement rate, b/w. Similarly, for match-related parameters, k, ξ , ϕ , λ^{P} and λ^{T} , we target labor market statistics including the fraction of temporary workers $\frac{n^{T}}{1-u}$, the job-finding rate, $\alpha^{w}(\theta)$, the unemployment rate u, and the job-to-job transitions for permanent and temporary workers $(J - J^{T} \text{ and } J - J^{P})$. Regarding productivity, however, there is no direct measure on match-quality z. We can only observe the total productivity in the data which leads to a potential under-identification problem. To tackle this issue, we run a wage regression that links wages, productivity, and the type of contract held. We target the coefficients of the wage regression plus the wage ratio between permanent workers and temporary workers to calibrate parameters related to productivity.

Specifically, we assume that the time-varying component of productivity y is firm-specific. Consequently, differences among workers' wages within a firm will be the result of working under a different contract or of having a different match-specific quality. We then posit that the (log) wage of worker i of firm j at time t is given by:

(20)
$$\ln(w_{ijt}) = a_0 + a_1 \ln(ALP_{jt}) + a_2 \chi_{ijt} + \varepsilon_{ijt}$$

where ALP_{jt} is an establishment's average labor productivity – output divided by total hours – and χ_{ijt} is an indicator variable describing a worker's temporary status. This is the equation we estimate based on the WES data. The analogous equation to (20) used in our model simulation is the following. Since we view one firm *j* hires a large number of workers, the average productivity ALP_{jt} is the sum of the time-varying component y_{jt} plus an expectation of the match-specific

of notice are required in dismissing permanent workers in most Canadian provinces. The exact length varies by tenure duration. In addition, according to Canada labour Code, severance pay is mandated for workers above certain tenure duration. See the Appendix for more details on employment protection in Canada.

Parameter	Value	Moment	Model	Data	Data
			Moment	Moment	Source
f	0.18	$\frac{n^T}{1}$	0.094	0.126	LFS
k	1.41	$\alpha^{w}(\theta)$	0.937	0.919	Zhang (2008)
b	0.47	$\frac{f}{w^P}$	0.182	0.182	WES
ξ	5.76	$\frac{w}{b}{w}$	0.475	0.550	WES
σ_{z}	0.28	и	0.136	0.084	LFS
μ_y	340	$\frac{w^P}{w_T^T}$	1.151	1.329	LFS
σ_y	1.11	$J - \tilde{J}^P$	0.116	0.097	SLID
$\sigma_y \ \lambda^P$	0.28	$J - J^T$	0.275	0.256	SLID
λ^T	0.27	a_1	0.174	0.159	WES
ϕ	0.17	a_2	0.117	0.193	WES

Table 2: Parameter Values and Targeted Moments

Note: The table reports parameter values for the model's structural parameters (first two columns) as well as the moments targeted (data and model value). The last column reports the source for the empirical moment. LFS is Labor Force Survey, WES is Workplace Employment Survey and SLID is Survey of Labour and Income Dynamics.

productivity *z* at time *t*. The wage regression in the model will become

(21)
$$\ln\left(\hat{w}_{ijt}\right) = \hat{a}_0 + E\left(z\right) + \hat{a}_1 \ln\left(ALP_{jt}\right) + \hat{a}_2 \chi_{ijt} + \hat{\varepsilon}_{ijt}.$$

Disturbances in this regression will be interpreted as deviations of the match-specific quality for a given match relative to its mean match-specific value. Our sample of the WES dataset covers the years 2001 to 2006. We estimate equation (20) for each year and calculate the average of coefficients a_1 and a_2 as the moments that the model tries to match. The last two columns in Table 2 summarizes the targeted moments, their values, and the sources of those values.

The second column in Table 2 shows the resulting parameter values from the calibrating procedure. The implied model moments are shown in the fourth column. They are more or less in line with their empirical counterparts with the exception of the unemployment rate, which the model over-predicts.

4.3 The Rise in Aggregate Uncertainty Post-2009

Our focus is to understand the change in aggregate uncertainty after the Great Recession and its impact on the labor market. In the model, we can view one firm *j* as hiring a large number of workers of different match qualities. If we assume that the time-varying component of productivity *y* is firm-specific, then each match in the firm yields the output $z_i + y_{jt}$, where z_i is the match-quality which is fixed within the match. The total output within firm *j* is the aggregation of all the matches as $\int_i z_i di + y_{jt} = E(z) + y_{jt}$. In other words, the firm's output consists of a permanent component which is the same across all firms and a transitory component which draws from the same distribution *F*. The volatility of this transitory component is our measure of aggregate uncertainty. However, in the data, we do not have a long time series on the firm-level output. The best we can get is the time series across industries. Fortunately, our theory allows us to interpret matches occurring at the industry level.

Given this interpretation, we assemble output across 217 business sectors from Statistics Canada Table 36100480. The output in each sector is at the most detailed level of the North American Industry Classification System (NAICS) — sectors are given at the four or five digits of NAICS. We calculate standard deviations of cross-sectoral output from 1997 to 2015 and plot them in the first panel of Figure 4. As is evident from the figure, there is a clear increase in the uncertainty after 2008. The cross-sectoral output dispersion was 1.14 on average before 2008 and jumped to 1.22 after 2008, an increase of 7.1%. This finding is consistent with the existing literature on uncertainty measurement. A large and growing literature has documented that the level of uncertainty over the period 2007 – 2012 has heightened as suggested by various measures at the macro and micro levels. See, for

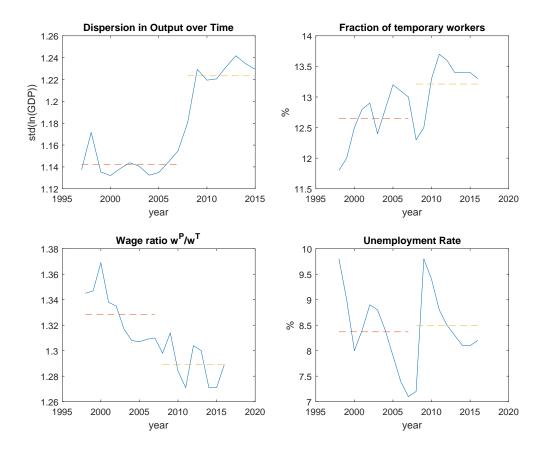


Figure 4: Aggregate uncertainty and labor market outcomes before and after crisis example, Bloom (2014) which surveys the literature. Jurado et al. (2015) show that uncertainty is large and persistent during the Great Recession.

Information on temporary employment is available from the year 1997 in the Labor Force Survey (LFS). The LFS data is monthly, we calculate the annual share of temporary employment by using yearly average of monthly temporary employment and the yearly average of monthly total employment. The annual share of temporary employment in total employed workers rose from 12.6% before 2008 to 13.2% after 2008. The wage difference between permanent workers and temporary workers measured as the mean hourly wage ratio dropped from 1.33 to 1.29, a 3% decrease. The average unemployment rate slightly went up from 8.4% to 8.5%. Table 3 documents these facts.

We now use our model to explain these labor market facts. We show that the

	σ_y	$\frac{n^T}{1-u}$	$\frac{w^P}{w^T}$	и
Before 2008	1.14	12.6	1.329	8.4
After 2008	1.22	13.2	1.289	8.5
Ratio	1.07	1.05	0.97	1.01
Source	Table 36100480	LFS	LFS	LFS

Table 3: Changes in labor market outcomes

Note: The table compares several variables obtained from data for two subperiods: 1997-2007 (first row) vs. 2009-2015 (second row). Ratio represents the ratio of values in the second period relative to the first. The four variables are the standard deviation of the time-varying component of productivity (σ_y), the fraction of temporary workers ($n_T/(1-u)$), the permanent contract wage premium (w^P/w^T), and the unemployment rate u. *LFS* refers to the Labor Force Survey and Table 36100480 is provided by Statistics Canada.

rise in aggregate uncertainty provides an explanation for the trend changes. The benchmark economy is calibrated based on pre-2008 data. If we keep other structural parameters the same and increase the volatility of *y* by 7.1% as we observe in the data, the model economy resemble the steady state after the crisis. Table 4 shows the results. Comparing to the data, our model is consistent with observed labor market facts. The fraction of temporary workers rises while the wage ratio drops. The unemployment rate increases slightly. These patterns are robust if we change the output dispersion from 70% of its benchmark value to a factor 1.5 of the benchmark value. The first three rows in Table 5 shows the trends in response to the rise in σ_y . In summary, our model rationalizes the changes in the labor market due to a rise in uncertainty.

A concern is that the observed rise in the fraction of temporary employment may be the result of a composition effect. Not all sectors have the same fraction of temporary workers (e.g. Hospitality Industries vs Financial Services). The financial crisis and the recession that ensued may have caused a sector reallocation

		Data				M	lodel	
	σ_y	$\frac{n^T}{1-u}$	$rac{w^P}{w^T}$	и	σ_y	$\frac{n^T}{1-u}$	$rac{w^P}{w^T}$	и
Before 2008	1.14	12.6	1.329	8.4	1.11	9.4	1.151	13.6
After 2008	1.22	13.2	1.289	8.5	1.19	10.3	1.144	13.9
Ratio	1.07	1.05	0.97	1.01	1.07	1.09	0.99	1.02

Table 4: Changes in labor market outcomes

Note: The table compares model outcome across the two time periods to the empirical analogs. The variables are the same as those in Table 3 and first four columns report the same statistics. The last four columns report the model variables. *Ratio* is calculated as the ratio of the variable value in the second period to the variable value in the first period.

during which sectors with a higher fraction of temporary contracts became larger. To check for this possibility we perform a within-between decomposition. The goal of the decomposition is to divide the rise in temporary contracts (as a fraction of employment) into changes within sectors and across sectors. Using the Labour Force Survey (LFS) micro-data we calculate the share of temporary employment for 17 sectors. We then decompose the change in the aggregate share of temporary employment into contribution from within sectors and between sectors. We calculate this decomposition for the change of temporary employment from 1997 to 2015.¹⁷ We find that the between-sector contribution is about 20% of the total increase in the share of temporary employment. This implies that data does not support the hypothesis that structural change plays a major role in the increase of temporary employment after 2008.

What is the reason that an increase in the dispersion of y has this particular impact on dual labor markets? First of all, as we have shown in Section 3, the

¹⁷Let the share of temporary employment be $\frac{T}{E}$. Then $\frac{T}{E} = \sum_{i=1}^{N} \frac{T_i}{E} = \sum_{i=1}^{N} \frac{E_i}{E} \cdot \frac{T_i}{E_i}$. Let $\omega_i = \frac{E_i}{E}$ and $\tau_i = \frac{T_i}{E_i}$. Then $\frac{T_t}{E_t} = \sum_{i=1}^{N} \omega_{it} \tau_{it}$. The change is $\Delta \frac{T_t}{E_t} = \sum_{i=1}^{N} (\omega_{it} \tau_{it} - \omega_{it-1} \tau_{it-1}) = \sum_{i=1}^{N} \Delta \omega_{it} \cdot \tau_{it} + \sum_{i=1}^{N} \omega_{it-1} \cdot \Delta \tau_{it}$.

	$0.7 imes \sigma_y$	Benchmark	$1.071 imes \sigma_y$	$1.5 imes \sigma_y$
$\frac{\frac{n^{T}}{1-u}}{\frac{w^{P}}{w^{T}}}$	5.7	9.4	10.4	14.7
$\frac{w^P}{w^T}$	1.187	1.151	1.149	1.102
u	11.8	13.6	14.3	15.2
\bar{z}	-338.54	-338.43	-338.41	-338.30

Table 5: Effects of σ_{y} on the labor market

Note: The table shows the effects of a larger change in the productivity standard deviation σ_y . The middle two columns correspond to the 1997-2007 calibration (*Benchmark*) and the column labeled $1.071 \times \sigma_y$ is the results with the approximate increase in dispersion observed in the data. The first and last columns represent outcomes with a 30% drop and a 50% rise in σ_y respectively.

equilibrium can be characterized by simple cutoff rules illustrated in Figure 1. A rise in the dispersion of *y* increases both the upside risk and the downside risk for each job. But the effect is asymmetric across different contracts. For low quality matches, only the upside risk matters; it the productivity drawn next period is low, the worker is fired: getting more draws on low productivity values does not matter much since they never meet the threshold anyway. The probability of getting a high productivity draw of y is large and the firm is more likely to retain the worker. Therefore, for low quality matches, the separation cutoff will be lowered in response to an increase in the upside risk reflecting a higher chance of retention. This situation is illustrated in Figure 5 panel (a). In contrast, for high quality matches, the downside risk dominates. This is because the separation cutoff point is low. If the risk of getting low values of y is high, the match will more likely be destroyed. The low bar on separation also implies that an increase in the probability of high values of y is less an issue. Consequently, the separation cutoff will rise reflecting avoidance of downside risk for high quality matches which is shown in panel (b) of Figure 5. This asymmetric effect on different match qualities makes the firing rule flatter as Figure 5 shows. As low quality matches are typically

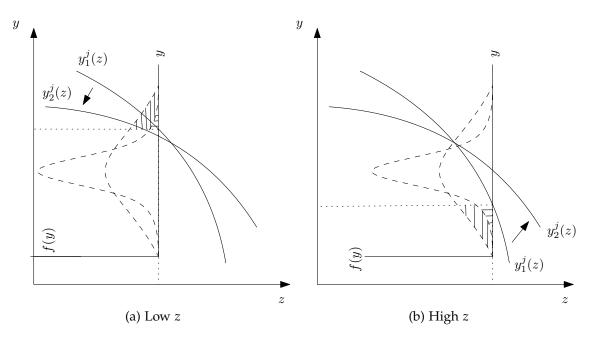


Figure 5: Asymmetric effects of σ_{ν} on firing rules

in temporary contracts while high quality matches form permanent contracts, it implies that y^T will go down and y_0^p will go up when facing higher σ_y . In other words, the firm will be more likely to promote temporary workers which increases the value of temporary jobs. The firm will fire permanent workers more often which decreases the value of permanent jobs. As Figure 6 shows, a rise in output dispersion will induce more temporary contracts offered, and \bar{z} going up. This potentially explains why the fraction of temporary workers becomes bigger.

The above intuition can be verified by looking at the decision rules in our quantitative experiments. In our simulation, the differences in firing rules, y_0^p and y^T are very small, less than 0.2% of changes. It is not easy to visualize these small differences as Figure 7 depicts. Instead, we plot the percentage changes in y_0^p and y^T due to the increase in σ_y . Figure 8 shows the resulting changes of 7.1% and 50% increase in the standard deviation of y relative to the benchmark economy respectively. We can see that when σ_y goes up, both firing rules will go down for low values of z, (as $\Delta y^j \equiv y^j (\sigma_i) - y^j (\sigma_{Benchmark}) < 0$) and then go up for high

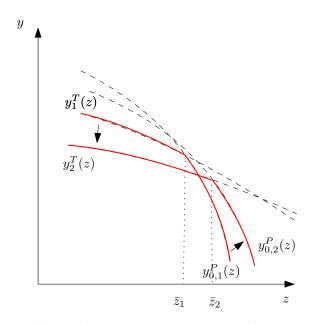


Figure 6: Effect of an increase in σ_y on different contracts

values of z (as $\Delta y^j > 0$). This is consistent with the asymmetric effect of output dispersion on different match qualities, and hence, different types of contract. The last row of Table 5 confirms that the cutoff \bar{z} is increasing with respect to σ_y .

This asymmetry of the impact of uncertainty on contract types affects job turnover. First, permanent workers are more likely to lose job due to a drop in y_0^p . Second, temporary workers are less likely to be fired as y^T moves up. Third, unemployed workers are more likely to find temporary jobs than permanent jobs. Transition matrices listed in Table 6 confirm these findings. The probability of permanent workers losing jobs grows from 0.092 to 0.131 when σ_y goes up from 30% less of the benchmark value to 50% more of the value. At the same time, the probability of permanent workers to keep own jobs or find other permanent jobs drops from 0.891 to 0.819. The firms promote temporary workers more often. The promotion rate for temporary workers rises from 69.9% to 71.4%. In contrast, the odds that temporary workers become unemployed fall from 25.5% to 20.9%. The probability that an unemployed worker finds a temporary job rises from 28.9% to 51.8% while the chance of finding a permanent job diminishes from 46.6% to 27.4%. Our

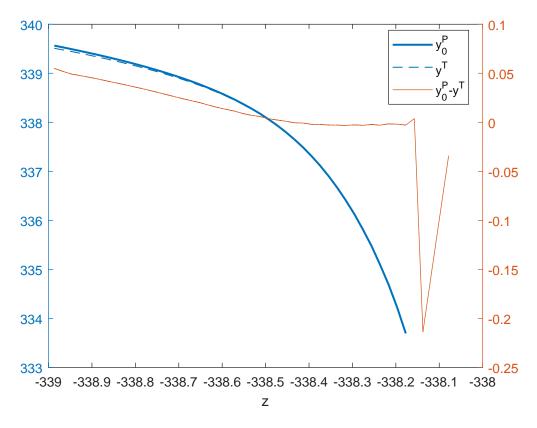


Figure 7: Firing rules in the benchmark economy

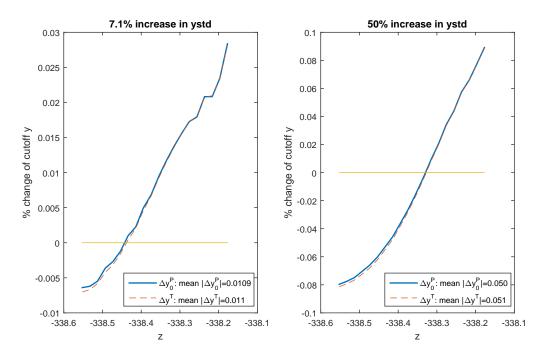


Figure 8: Differences in firing rules due to increases in σ_y

	Variables			$0.7 imes\sigma_y$			$1.0 imes \sigma_y$		
	n^P	n^T	и	n^P	n^T	и	n^P	n^T	и
n^P	π^{PP}	π^{TP}	π^{UP}	0.891	0.699	0.466	0.858	0.703	0.399
n^T	π^{PT}	π^{TT}	π^{UT}	0.017	0.046	0.289	0.029	0.059	0.392
и	π^{PU}	π^{TU}	π^{UU}	0.092	0.255	0.245	0.113	0.237	0.209
	Variables		$1.071 imes \sigma_y$			$1.5 imes \sigma_y$			
	n^P	n^T	и	n^P	n^T	u	n^P	n^T	и
n^P	π^{PP}	π^{TP}	π^{UP}	0.848	0.699	0.381	0.819	0.714	0.274
n^T	π^{PT}	π^{TT}	π^{UT}	0.032	0.063	0.412	0.051	0.078	0.518
и	π^{PU}	π^{TU}	π^{UU}	0.120	0.238	0.207	0.131	0.209	0.208

Table 6: Transition matrices across different σ_{y}

quantitative results also show that it's easier for unemployed workers to find positions as output dispersion arises. Workers are less likely to remain unemployed; in other words the job finding rate rises. This is due to the general equilibrium effect through which, after the rise in uncertainty, there are more vacancies per unemployed person. The job finding rate ascends from 87% (when uncertainty is 70% of the benchmark case), to 94% when uncertainty is at its benchmark values,i to 0.95 when uncertainty is 50% higher than its benchmark value. Because the fraction of permanent workers accounts for more than 75% of the total labor force, the outflow from permanent jobs to unemployment dominates inflows from retention of temporary workers and from unemployment to employment. As a result, total unemployment rises slightly.

The wage premium that a permanent worker earns diminishes because the rise in \bar{z} changes the productivity composition of different types of contracts. According to wage equations (14) and (17), the average wages for permanent workers and temporary workers are calculated from taking the expectations conditional on contract types, i.e., conditional on \bar{z} . Since σ_y rises, \bar{z} rises too; not just the mass of temporary workers increases, but the aggregate match-quality also rises. This is because some high-quality matches counted as permanent jobs before become fixed-term contracts. A higher aggregate match-quality results in a higher average wage of temporary jobs. On the other hand, although individual match-quality increases under permanent contracts as the bar offering such contracts is higher, the measure of permanent workers goes down which offsets the rise in quality. The aggregate match-quality for permanent jobs remains more or less the same. As a result, the composition effect of match-quality changes is responsible for a lower wage ratio, w^P/w^T . Figure 9 plots the simulated wage distributions of permanent jobs and temporary jobs under different scenarios of σ_y . It is clear that the wage distribution of temporary jobs shifts to the right as productivity dispersion rises which reflects the increase in the conditional expectation of *z*. However, the horizontal movement in the distribution of permanent jobs is not significant compared to the case of temporary jobs. The wage premium drops.

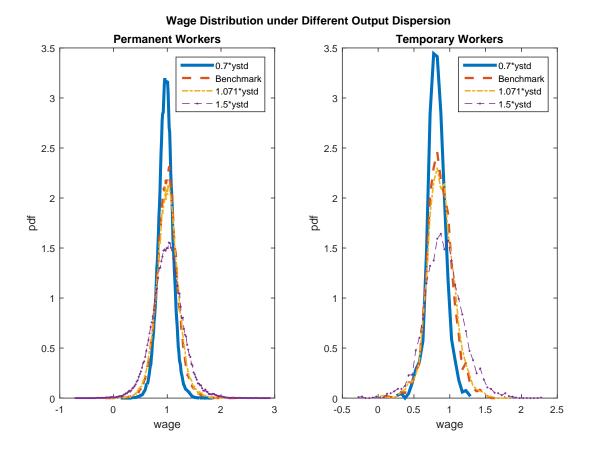


Figure 9: Effect of σ_v on wage distributions

5 Conclusion

We build a search and matching model incorporating endogenous separations and on-the-job search to explain the choice between temporary and permanent contracts. We show that the initial labor contract is determined by the matchquality, which varies across worker-firm pairs and is revealed when the firm and the worker meet. Firms offer permanent contracts to "good" matches despite a higher severance cost may involve, as they risk losing the worker should they offer them a temporary contract. This risk results from the different on-the-job search behavior by the two types of workers: temporary workers search more often than permanent workers. Not-so-good matches are given a temporary contract to take advantage of low firing costs. After the temporary contract expires, the firm can decide to dismiss or promote the temporary worker to a permanent one. We use our model to examine the recent changes in the labor market trend since the Great Recession. In Canada, the share of temporary workers has increased significantly. But at the same time, the wage inequality measured as the ratio of the wage a permanent worker earns relative to that of a temporary worker has dropped. Yet, unlike other OECD countries, such as Spain, experienced a big jump in unemployment (see Bentolila et al., 2012), the total unemployment has increased slightly. We find that the interplay among endogenous hiring, firing and promotion amid heightened uncertainty can explain these facts. The rise in uncertainty since the Great Recession generates asymmetric effects on temporary and permanent employment. It moves up the value of temporary jobs but brings down the value of permanent jobs. The job turnovers for different types of workers change accordingly. As a result, changes in the composition of labor markets and the productivity distribution help explain these salient facts.

This paper identifies the impact of uncertainty on a labor market through a composition channel on temporary and permanent employment. As the literature points out, uncertainty can affect the labor market through other channels (see Bloom, 2009; Leduc and Liu, 2016; Basu and Bundick, 2017; and Schaal, 2017). An extension of the current framework will be useful to address labor market dynamics over the business cycle. We leave this for future research.

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A Appendix: Additional Material

A.1 The Value of Employment and Unemployment

This section states the value functions for workers across the different employment and unemployment states.

The value for an unemployed agent consists of a flow of unemployment benefits (net of taxes) this period, plus a continuation value. The continuation ends in two possible states: either the individual finds a new job or continues his search.

(22)
$$U = b - \tau + \beta \left(1 - \alpha^{w}(\theta)\right) U + \beta \alpha^{w}(\theta) \int_{z_{min}}^{z_{max}} V^{N}(z) dG(z) dG(z)$$

For each employed individual the employment values are determined by a wage (that is contingent on the contract) and a continuation value that comprises two states: an on-the-job offer and a decision at the separation stage from the current employer. The wages for the three different permanent worker states (existing, new hire, recently promoted) are:

$$V^{P}(y,z) = w^{P}(y,z) - \tau +\beta\lambda^{P}\alpha^{w}(\theta) \int_{z_{min}}^{z_{max}} \max\left\{V^{N}(\tilde{z}), E_{y}\left[\max\left(V^{P}(y,z), U\right)\right]\right\} dG(\tilde{z}) +\beta(1-\lambda^{P}\alpha^{w}(\theta))E_{y}\left[\max\left(V^{P}(y,z), U\right)\right].$$
(23)

$$V_{0}^{P}(y,z) = w_{0}^{P}(y,z) - \tau +\beta\lambda^{P}\alpha^{w}(\theta) \int_{z_{min}}^{z_{max}} \max\left\{V^{N}(\tilde{z}), E_{y}\left[\max\left(V^{P}(y,z),U\right)\right]\right\} dG(\tilde{z}) +\beta(1-\lambda^{P}\alpha^{w}(\theta))E_{y}\left[\max\left(V^{P}(y,z),U\right)\right].$$
(24)

$$V^{R}(y,z) = w^{R}(y,z) - \tau +\beta\lambda^{P}\alpha^{w}(\theta) \int_{z_{min}}^{z_{max}} \max\left\{V^{N}(\tilde{z}), E_{y}\left[\max\left(V^{P}(y,z), U\right)\right]\right\} dG(\tilde{z}) +\beta(1-\lambda^{P}\alpha^{w}(\theta))E_{y}\left[\max\left(V^{P}(y,z), U\right)\right].$$
(25)

For a temporary worker the structure of the equation determining the value of being employed is identical. The flow income is a smaller wage and the probability of an outside offer is driven by λ_T instead of λ_P .

$$V^{T}(y,z) = w^{T}(y,z) - \tau$$

+ $\beta\lambda^{T}\alpha^{w}(\theta)\int_{z_{min}}^{z_{max}} \max\left\{V^{N}(\tilde{z}), E_{y}\left[\max\left(V^{R}(y,z), U\right)\right]\right\} dG(\tilde{z})$
(26) + $\beta\left(1 - \lambda^{T}\alpha^{w}(\theta)\right)E_{y}\left[\max\left(V^{R}(y,z), U\right)\right].$

A.2 The Values of a Vacancy and a Filled Job

To calculate the value of an unfilled vacancy, the firm takes into account the distribution of workers across the different contracts. Recall that u is the fraction of unemployed workers, n^p is the fraction of permanent workers, and n^T is the fraction of temporary workers. Notice that $u + n^p + n^T = 1$. The value function for a vacant job is,

$$Q = -k + \beta \alpha^{f}(\theta) u \int_{z_{min}}^{z_{max}} \max \left\{ E_{y} \left[\max \left(J_{0}^{P}(y,z), Q \right) \right], E_{y} \left[\max \left(J^{T}(y,z), Q \right) \right] \right\} dG(z) + \beta \alpha^{f}(\theta) n^{P} \int_{\hat{z} \in [z_{min}, z_{max}]} \int_{z \in B^{P}(\hat{z})} \max \left\{ \begin{array}{c} E_{y} \left[\max \left(J_{0}^{P}(y,z), Q \right) \right], \\ E_{y} \left[\max \left(J^{T}(y,z), Q \right) \right], \end{array} \right\} dG(z) dG(\hat{z}) + \beta \alpha^{f}(\theta) n^{T} \int_{\hat{z} \in [z_{min}, \hat{z}]} \int_{z \in B^{T}(\hat{z})} \max \left\{ \begin{array}{c} E_{y} \left[\max \left(J_{0}^{P}(y,z), Q \right) \right], \\ E_{y} \left[\max \left(J^{T}(y,z), Q \right) \right], \end{array} \right\} dG(z) dG(\hat{z}) + (\beta T (1 - \alpha^{f}(\theta)) Q, \end{array} \right\}$$

The firm pays a cost k and hopes to match with a worker so that production can begin. The chance that a meeting results in a match depends on the type of worker the firm meets. Hence meeting a permanent worker yields a different payoff than meeting an unemployed individual. Note that in equilibrium, because of our assumption that firms are free to enter and post vacancies, Q = 0 is the only value that satisfies the previous equation.

$$J^{P}(y,z) = y + z - w^{P}(y,z) + \beta \left(1 - \lambda^{P} \alpha^{w}(\theta)\right) E_{y} \left[\max\left(J^{P}(y,z), Q - f\right)\right] + \beta \lambda^{P} \alpha^{w}(\theta) \left\{\rho^{P}(z) Q + \left(1 - \rho^{P}(z)\right) E_{y} \left[\max\left(J^{P}(y,z), Q - f\right)\right]\right\}.$$

$$J_{0}^{P}(y,z) = y + z - w_{0}^{P}(y,z) + \beta \left(1 - \lambda^{P} \alpha^{w}(\theta)\right) E_{y} \left[\max\left(J^{P}(y,z), Q - f\right)\right] + \beta \lambda^{P} \alpha^{w}(\theta) \left\{\rho^{P}(z) Q + \left(1 - \rho^{P}(z)\right) E_{y} \left[\max\left(J^{P}(y,z), Q - f\right)\right]\right\}.$$

$$J^{T}(y,z) = y + z - w^{T}(y,z) + \beta \left(1 - \lambda^{T} \alpha^{w}(\theta)\right) E_{y} \left[\max\left(J^{R}(y,z),Q\right)\right] + \beta \lambda^{T} \alpha^{w}(\theta) \left\{\rho^{T}(z) Q + \left(1 - \rho^{T}(z)\right) E_{y} \left[\max\left(J^{R}(y,z),Q\right)\right]\right\}.$$

$$J^{R}(y,z) = y + z - w^{R}(y,z) - c + \beta \left(1 - \lambda^{P} \alpha^{w}(\theta)\right) E_{y} \left[\max\left(J^{P}(y,z), Q - f\right)\right]$$

(31)
$$+\beta \lambda^{P} \alpha^{w}(\theta) \left\{\rho^{P}(z) Q + \left(1 - \rho^{P}(z)\right) E_{y} \left[\max\left(J^{P}(y,z), Q - f\right)\right]\right\}.$$

A.3 Conditional Switching Probabilities

Let ρ^{ij} denote the transition probability of a worker from type *i* to type *j* conditional on finding a firm. In particular, the expression of ρ^{ij} is given by

$$\begin{split} \rho^{TT} &= \frac{1}{G\left(\bar{z}\right)} \int_{z_{min}}^{\bar{z}} \int_{z \in B^{T}(\hat{z}) \cap [z_{min},\bar{z}]} \left(1 - F\left(y^{T}\left(z\right)\right)\right) dG\left(z\right) dG\left(\hat{z}\right) \\ &= \frac{1}{G\left(\bar{z}\right)} \int_{z_{min}}^{\bar{z}} \int_{z \in [z^{T}(\hat{z}), z_{max}] \cap [z_{min},\bar{z}]} \left(1 - F\left(y^{T}\left(z\right)\right)\right) dG\left(z\right) dG\left(\hat{z}\right), \end{split}$$

$$\begin{split} \rho^{TP} &= \frac{1}{G\left(\bar{z}\right)} \int_{z_{min}}^{\bar{z}} \int_{z \in B^{T}(\hat{z}) \cap [\bar{z}, z_{max}]} \left(1 - F\left(y_{0}^{P}\left(z\right)\right)\right) dG\left(z\right) dG\left(\hat{z}\right) \\ &= \frac{1}{G\left(\bar{z}\right)} \int_{z_{min}}^{\bar{z}} \int_{z \in [z^{T}(\hat{z}), z_{max}] \cap [\bar{z}, z_{max}]} \left(1 - F\left(y_{0}^{P}\left(z\right)\right)\right) dG\left(z\right) dG\left(\hat{z}\right), \end{split}$$

$$\begin{split} \rho^{PT} &= \int_{z_{min}}^{z_{max}} \int_{z \in B^{P}(\hat{z}) \cap [z_{min}, \bar{z}]} \left(1 - F\left(y^{T}(z)\right) \right) dG(z) dG(\hat{z}) \\ &= \int_{z_{min}}^{z_{max}} \int_{z \in [z^{P}(\hat{z}), z_{max}] \cap [z_{min}, \bar{z}]} \left(1 - F\left(y^{T}(z)\right) \right) dG(z) dG(\hat{z}) \,, \end{split}$$

$$\begin{split} \rho^{PP} &= \int_{z_{min}}^{z_{max}} \int_{z \in B^{P}(\hat{z}) \cap [\bar{z}, z_{max}]} \left(1 - F\left(y_{0}^{P}(z)\right) \right) dG(z) dG(\hat{z}) \\ &= \int_{z_{min}}^{z_{max}} \int_{z \in [z^{P}(\hat{z}), z_{max}] \cap [\bar{z}, z_{max}]} \left(1 - F\left(y_{0}^{P}(z)\right) \right) dG(z) dG(\hat{z}) \,, \end{split}$$

B Appendix: Proof of Propositions

Proof of Proposition 1. Replacing U with equation (22), we can rewrite equation (5) as

$$S^{P}(y,z) = y + z - b + \beta \left(1 - \lambda^{P} \alpha^{w}(\theta) \rho^{P}(z)\right) E_{y} \left[\max\left(S^{P}(y,z),0\right)\right] + \phi \beta \lambda^{P} \alpha^{w}(\theta) \int_{B^{P}(z)} S^{N}(\tilde{z}) dG(\tilde{z}) (32) \qquad -\phi \beta \alpha^{w}(\theta) \int_{z_{min}}^{z_{max}} S^{N}(\tilde{z}) dG(\tilde{z}) + \left[1 - \beta \left(1 - \lambda^{P} \alpha^{w}(\theta) \rho^{P}(z)\right)\right] f.$$

Since $\partial S^{P}/\partial y = 1 > 0$, if the cutoff $y^{P}(z)$ exists such that $S^{P}(y^{P}(z), z) = 0$, then for any $y > y^{P}(z)$, $S^{P}(y, z) > 0$, i.e.

(33)
$$E_{y}\left[\max\left(S^{P}\left(y,z\right),0\right)\right] = \int_{y^{P}(z)}^{y_{max}} S^{P}\left(y,z\right) dF\left(y\right).$$

Because $\partial^2 S^P / \partial y \partial z = 0$, we can write $S^P(y, z) = y + \varphi(z)$. The integral in (33) is then

$$\int_{y^{P}}^{y_{max}} S^{P}(y,z) \, dF(y) = \int_{y^{P}}^{y_{max}} y + \varphi(z) \, dF(y) \,,$$

= $(y + \varphi(z)) F(y) |_{y^{P}}^{y_{max}} - \int_{y^{P}}^{y_{max}} F(y) \, dy.$

For any $z \in Z$, $S^{P}(y^{P}, z) = 0$ implies $y^{P} = -\varphi(z)$. Substitute $\varphi(z)$ with $-y^{P}$, the expression of the integral is

(34)
$$\int_{y^{P}}^{y_{max}} S^{P}(y,z) \, dF(y) = \int_{y^{P}}^{y_{max}} \left[1 - F(y)\right] dy.$$

To pin down y^{p} , we need to solve the equation $S^{p}(y^{p}, z) = 0$, thus

(35)
$$y^{P} + z + \beta \left(1 - \lambda^{P} \alpha^{w} \left(\theta\right) \rho^{P} \left(z\right)\right) \int_{y^{P}}^{y_{max}} \left[1 - F\left(y\right)\right] dy$$
$$+ \phi \beta \lambda^{P} \alpha^{w} \left(\theta\right) \int_{B^{P}(z)} S^{N} \left(\tilde{z}\right) dG \left(\tilde{z}\right) - \phi \beta \alpha^{w} \left(\theta\right) \int_{z_{min}}^{z_{max}} S^{N} \left(\tilde{z}\right) dG \left(\tilde{z}\right)$$
$$= b - \left[1 - \beta \left(1 - \lambda^{P} \alpha^{w} \left(\theta\right) \rho^{P} \left(z\right)\right)\right] f.$$

Similarly, the value of the new permanent worker can be written as

$$S_{0}^{P}(y,z) = y + z - b + \beta \left(1 - \lambda^{P} \alpha^{w}(\theta) \rho^{P}(z)\right) E_{y} \left[\max\left(S^{P}(y,z),0\right)\right] + \phi \beta \lambda^{P} \alpha^{w}(\theta) \int_{B^{P}(z)} S^{N}(\tilde{z}) dG(\tilde{z}) - \phi \beta \alpha^{w}(\theta) \int_{z_{min}}^{z_{max}} S^{N}(\tilde{z}) dG(\tilde{z}) - \beta \left(1 - \lambda^{P} \alpha^{w}(\theta) \rho^{P}(z)\right) f.$$
(36)

The value of new promotion in 7 is

$$S^{R}(y,z) = y + z - c - b + \beta \left(1 - \lambda^{P} \alpha^{w}(\theta) \rho^{P}(z)\right) E_{y} \left[\max\left(S^{P}(y,z),0\right)\right] + \phi \beta \lambda^{P} \alpha^{w}(\theta) \int_{B^{P}(z)} S^{N}(\tilde{z}) dG(\tilde{z}) (37) \qquad - \phi \beta \alpha^{w}(\theta) \int_{z_{min}}^{z_{max}} S^{N}(\tilde{z}) dG(\tilde{z}) - \beta \left(1 - \lambda^{P} \alpha^{w}(\theta) \rho^{P}(z)\right) f.$$

The value of the temporary worker in (8) is

$$S^{T}(y,z) = y + z - b + \beta \left(1 - \lambda^{T} \alpha^{w}(\theta) \rho^{T}(z)\right) E_{y} \left[\max\left(S^{R}(y,z),0\right)\right] + \phi \beta \lambda^{T} \alpha^{w}(\theta) \int_{B^{P}(z)} S^{N}(\tilde{z}) dG(\tilde{z}) - \phi \beta \alpha^{w}(\theta) \int_{z_{min}}^{z_{max}} S^{N}(\tilde{z}) dG(\tilde{z}).$$

Following the same argument for the condition $S^{P}(y^{P}, z) = 0$, the above equations yield the cut-off values by solving:

(39)

$$y_{0}^{P} + z + \beta \left(1 - \lambda^{P} \alpha^{w} \left(\theta\right) \rho^{P} \left(z\right)\right) \int_{y^{P}}^{y_{max}} \left[1 - F\left(y\right)\right] dy$$

$$+ \phi \beta \lambda^{P} \alpha^{w} \left(\theta\right) \int_{B^{P}(z)} S^{N} \left(\tilde{z}\right) dG \left(\tilde{z}\right) - \phi \beta \alpha^{w} \left(\theta\right) \int_{z_{min}}^{z_{max}} S^{N} \left(\tilde{z}\right) dG \left(\tilde{z}\right)$$

$$= b + \beta \left(1 - \lambda^{P} \alpha^{w} \left(\theta\right) \rho^{P} \left(z\right)\right) f,$$

(40)

$$y^{R} + z + \beta \left(1 - \lambda^{P} \alpha^{w} \left(\theta\right) \rho^{P} \left(z\right)\right) \int_{y^{P}}^{y_{max}} \left[1 - F\left(y\right)\right] dy$$

$$+ \phi \beta \lambda^{P} \alpha^{w} \left(\theta\right) \int_{B^{P}(z)} S^{N} \left(\tilde{z}\right) dG \left(\tilde{z}\right) - \phi \beta \alpha^{w} \left(\theta\right) \int_{z_{min}}^{z_{max}} S^{N} \left(\tilde{z}\right) dG \left(\tilde{z}\right)$$

$$= b + \beta \left(1 - \lambda^{P} \alpha^{w} \left(\theta\right) \rho^{P} \left(z\right)\right) f + c,$$

and

(41)

$$y^{T} + z + \beta \left(1 - \lambda^{T} \alpha^{w} \left(\theta\right) \rho^{T} \left(z\right)\right) \int_{y^{R}}^{y_{max}} \left[1 - F\left(y\right)\right] dy$$

$$+ \phi \beta \lambda^{T} \alpha^{w} \left(\theta\right) \int_{B^{T}(z)} S^{N} \left(\tilde{z}\right) dG \left(\tilde{z}\right) - \phi \beta \alpha^{w} \left(\theta\right) \int_{z_{min}}^{z_{max}} S^{N} \left(\tilde{z}\right) dG \left(\tilde{z}\right)$$

$$= b.$$

Comparing equations (39) and (40) with equation (35), we get

$$y_0^P = y^P + f,$$

$$y^R = y^P + f + c.$$

Denote the right hand side of (35), (39), (40) and (41) by Γ_P , Γ_{P0} , Γ_R and Γ_T respectively. Notice that

$$b + \beta f + c > \Gamma_R > \Gamma_{P0} > \Gamma_T > \Gamma_P > b - \left[1 - \beta \left(1 - \lambda^P \alpha^w \left(\theta_{max}\right)\right)\right] f,$$

for any $\theta \in [\theta_{min}, \theta_{max}]$ and $z \in [z_{min}, z_{max}]$. Denote $\Lambda_P(y^P)$, $\Lambda_{P0}(y^P_0)$, $\Lambda_R(y^R)$ and $\Lambda_T(y^T)$ the left hand side of (35), (39), (40) and (41) respectively. Notice that for any θ , z and $j \in \{P, P0, R, T\}$

(42)
$$\Lambda_{j}\left(y_{max}\right) > y_{max} + z_{min} - \phi \beta \alpha^{w}\left(\theta\right) \int_{z_{min}}^{z_{max}} S^{N}\left(\tilde{z}\right) dG\left(\tilde{z}\right).$$

From equation (27), since

$$\begin{aligned} k &= \beta \alpha^{f}(\theta) \, u \int_{z_{min}}^{z_{max}} \max \left\{ E_{y} \left[\max \left(J_{0}^{p}(y,z), Q \right) \right], E_{y} \left[\max \left(J_{0}^{T}(y,z), Q \right) \right] \right\} dG(z) \\ &+ \beta \alpha^{f}(\theta) \, n^{P} \int_{\hat{z} \in [z_{min}, z_{max}]} \int_{z \in B^{P}(\hat{z})} \max \left\{ \begin{array}{c} E_{y} \left[\max \left(J_{0}^{p}(y,z), Q \right) \right], \\ E_{y} \left[\max \left(J_{0}^{p}(y,z), Q \right) \right], \end{array} \right\} dG(z) \, dG(\hat{z}) \\ &+ \beta \alpha^{f}(\theta) \, n^{T} \int_{\hat{z} \in [z_{min}, \hat{z}]} \int_{z \in B^{T}(\hat{z})} \max \left\{ \begin{array}{c} E_{y} \left[\max \left(J_{0}^{p}(y,z), Q \right) \right], \\ E_{y} \left[\max \left(J_{1}^{T}(y,z), Q \right) \right], \end{array} \right\} dG(z) \, dG(\hat{z}) \\ &< \beta \alpha^{f}(\theta) \, u \int_{z_{min}}^{z_{max}} \max \left\{ E_{y} \left[\max \left(J_{0}^{p}(y,z), Q \right) \right], E_{y} \left[\max \left(J^{T}(y,z), Q \right) \right] \right\} dG(z) \, dG(z) \\ &+ \beta \alpha^{f}(\theta) \, n^{P} \int_{\hat{z} \in [z_{min}, z_{max}]} \int_{z \in [z_{min}, z_{max}]} \max \left\{ \begin{array}{c} E_{y} \left[\max \left(J_{0}^{p}(y,z), Q \right) \right], \\ E_{y} \left[\max \left(J_{0}^{p}(y,z), Q \right) \right], \end{array} \right\} dG(z) \, dG(z) \, dG(\hat{z}) \\ &+ \beta \alpha^{f}(\theta) \, n^{T} \int_{\hat{z} \in [z_{min}, z_{max}]} \int_{z \in [z_{min}, z_{max}]} \max \left\{ \begin{array}{c} E_{y} \left[\max \left(J_{0}^{p}(y,z), Q \right) \right], \\ E_{y} \left[\max \left(J_{0}^{p}(y,z), Q \right) \right], \end{array} \right\} dG(z) \, dG(\hat{z}) \\ &= \beta \alpha^{f}(\theta) \int_{z_{min}}^{z_{max}} \max \left\{ E_{y} \left[\max \left(J_{0}^{p}(y,z), Q \right) \right], E_{y} \left[\max \left(J^{T}(y,z), Q \right) \right] \right\} dG(z) \, dG(z) \, dG(z) \\ &= \beta \alpha^{f}(\theta) \int_{z_{min}}^{z_{max}} \max \left\{ E_{y} \left[\max \left(J_{0}^{p}(y,z), Q \right) \right], E_{y} \left[\max \left(J^{T}(y,z), Q \right) \right] \right\} dG(z) \, dG(z)$$

replacing J_0^p and J^T with S_0^p and S^T by using the surplus rule and arranging terms yields

$$\int_{z} S^{N}(z) dG(z) = \int_{z} \mathbb{I}_{A} E_{y} \left[\max \left(S_{0}^{P}(y,z), 0 \right) \right] + (1 - \mathbb{I}_{A}) E_{y} \left[\max \left(S^{T}(y,z), 0 \right) \right] dG(z)$$

$$(43) > \frac{k}{(1 - \phi) \beta \alpha^{f}(\theta)}.$$

Substituting (43) into (42) generates

$$\Lambda_{j}(y_{max}) > y_{max} + z_{min} - rac{\phi}{1-\phi} heta_{max}k.$$

It is straightforward to check that

$$\Lambda_{j}(y_{min}) < y_{min} + z_{max} + \beta \int_{y_{min}}^{y_{max}} (1 - F(y)) dy$$

Therefore, if inequalities (9) and (10) hold, then

$$\Lambda_{j}\left(y_{max}\right) > \Gamma_{j} > \Lambda_{j}\left(y_{min}\right)$$
,

for all θ , *z* and *j*. Since

$$\frac{d\Lambda_{j}}{dy^{j}} = 1 - \beta \left(1 - \lambda^{j} \alpha^{w} \left(\theta \right) \rho^{j} \left(z \right) \right) \left(1 - F \left(y^{j} \right) \right) > 0,$$

i.e., it is monotone, by continuity, we can conclude that a unique solution y^j exists for equations (35), (40) and (41).

Proof of Lemma 1. Equations (32), (36) and (38) implicitly define operators $T^j : C_1 \rightarrow C_1$ for $j \in \{P, P0, R, T\}$ where C_1 denotes the set of bounded continuous functions

 $S : Y \times Z \rightarrow R$ with the sup norm. It is straightforward to show that T^j is a contraction mapping so that S^j is a fixed point. To check this, notice that

$$E_{y}\left[\max\left(S_{1}^{j}\left(y,z\right),0\right)\right] \leq E_{y}\left[\max\left(S_{2}^{j}\left(y,z\right),0\right)\right],$$

for any $S_1^j \leq S_2^j$. Therefore, $T^j S_1^j \leq T^j S_2^j$. Also

$$E_{y}\left[\max\left(S^{j}\left(y,z\right)+\varepsilon,0\right)\right] \leq E_{y}\left[\max\left(S^{j}\left(y,z\right)+\varepsilon,\varepsilon\right)\right] = E_{y}\left[\max\left(S^{j}\left(y,z\right),0\right)\right]+\varepsilon$$

implies that $T^{j}(S^{j} + \varepsilon) \leq \beta \varepsilon + T^{j}S^{j}$. Using Blackwell's sufficient conditions and the contraction mapping theorem, we can draw the conclusion.

Given T^j is the contraction mapping and S^j is the fixed point, if S^j is strictly increasing in z, then the worker will switch job only if $\tilde{z} > z$. Denote $z^P(z)$ the cutoff value when $S^N(z^P) = E_y[\max(S^P(y,z), 0)]$, and $z^T(z)$ the cutoff value when $S^N(z^T) = E_y[\max(S^R(y,z), 0)]$. Notice that $dz^P/dz = dz^T/dz = 1$. The surpluses can be rewritten as

$$S^{j}(y,z) = y + z - b + \beta \left[1 - \lambda^{P} \alpha^{w}(\theta) \left(1 - G \left(z^{P}(z) \right) \right) \right] \int_{y^{P}(z)}^{y_{max}} (1 - F(y)) dy + \phi \beta \lambda^{P} \alpha^{w}(\theta) \int_{z^{P}(z)}^{z_{max}} \left[\begin{array}{c} \mathbb{I}_{A} \int_{y_{0}^{P}(z)}^{y_{max}} (1 - F(y)) dy \\+ (1 - \mathbb{I}_{A}) \int_{y^{T}(\tilde{z})}^{y_{max}} (1 - F(y)) dy \end{array} \right] dG(\tilde{z}) - \phi \beta \alpha^{w}(\theta) \int_{z_{min}}^{z_{max}} \left[\begin{array}{c} \mathbb{I}_{A} \int_{y_{0}^{P}(\tilde{z})}^{y_{max}} (1 - F(y)) dy \\+ (1 - \mathbb{I}_{A}) \int_{y^{T}(\tilde{z})}^{y_{max}} (1 - F(y)) dy \end{array} \right] dG(\tilde{z}) - \beta \lambda^{P} \alpha^{w}(\theta) \left(1 - G \left(z^{P}(z) \right) \right) f + \mathbb{I}_{\{j=P\}} f - \mathbb{I}_{\{j=R\}} c,$$

for $j \in \{P, P0, R\}$ and

$$S^{T}(y,z) = y + z - b + \beta \left[1 - \lambda^{T} \alpha^{w}(\theta) \left(1 - G \left(z^{T}(z) \right) \right) \right] \int_{y^{R}(z)}^{y_{max}} (1 - F(y)) dy + \phi \beta \lambda^{T} \alpha^{w}(\theta) \int_{z^{T}(z)}^{z_{max}} \left[\begin{array}{c} \mathbb{I}_{A} \int_{y^{P}_{0}(\tilde{z})}^{y_{max}} (1 - F(y)) dy \\ + (1 - \mathbb{I}_{A}) \int_{y^{T}(\tilde{z})}^{y_{max}} (1 - F(y)) dy \end{array} \right] dG(\tilde{z}) (45) \qquad -\phi \beta \alpha^{w}(\theta) \int_{z_{min}}^{z_{max}} \left[\begin{array}{c} \mathbb{I}_{A} \int_{y^{P}_{0}(\tilde{z})}^{y_{max}} (1 - F(y)) dy \\ + (1 - \mathbb{I}_{A}) \int_{y^{T}(\tilde{z})}^{y_{max}} (1 - F(y)) dy \end{array} \right] dG(\tilde{z}).$$

Taking the derivative with respect to *z* yields

$$\frac{\partial S^{j}}{\partial z} = \Phi_{P}(z) - \beta \left[1 - \lambda^{P} \alpha^{w} \left(1 - G \left(z^{P}(z) \right) \right) \right] \left(1 - F \left(y^{P} \right) \right) y^{P'}(z) ,$$

where

$$\begin{split} \Phi_{P}(z) &= 1 - \beta \lambda^{P} \alpha^{w} G'\left(z^{P}(z)\right) z^{P'}(z) \\ &\times \left\{ \phi \left[\begin{array}{c} \mathbb{I}_{A} \int_{y_{0}^{P}(z^{P})}^{y_{max}} \left(1 - F(y)\right) dy \\ &+ (1 - \mathbb{I}_{A}) \int_{y^{T}(z^{P})}^{y_{max}} \left(1 - F(y)\right) dy \end{array} \right] - \int_{y^{P}(z)}^{y_{max}} \left(1 - F(y)\right) dy + f \right\} \\ &= 1 - \beta \lambda^{P} \alpha^{w} G'\left(z^{P}(z)\right) \left\{ \phi S^{N}\left(z^{P}(z)\right) - E_{y}\left[\max\left(S^{P}(y, z), 0\right)\right] + f \right\}. \end{split}$$

From (35), the implicit function theorem implies that

$$y^{P'}(z) = -\frac{\Phi_P(z)}{1 - \beta \left[1 - \lambda^P \alpha^w \left(1 - G(z^P(z))\right)\right] (1 - F(y^P))}.$$

Therefore, $\partial S^{j}/\partial z > 0$ if and only if $\Phi_{P}(z) > 0$. Since

$$\phi S^{N}\left(z^{P}\left(z\right)\right) < E_{y}\left[\max\left(S^{P}\left(y,z\right),0\right)\right]$$

from the definition of z^P , it is sufficient to have

$$1 - \beta \lambda^{P} \alpha^{w} G'\left(z^{P}\left(z\right)\right) f > 1 - \beta \lambda^{P} G' f > 0,$$

for all *z* so that $\Phi_P(z) > 0$.

Similarly,

$$\frac{\partial S^{T}}{\partial z} = \Phi_{T}(z) - \beta \left[1 - \lambda^{T} \alpha^{w} \left(1 - G \left(z^{T}(z) \right) \right) \right] \left(1 - F \left(y^{R} \right) \right) y^{R'}(z) ,$$

where

$$\Phi_{T}(z) = 1 - \beta \lambda^{T} \alpha^{w} G'\left(z^{T}(z)\right) \left\{\phi S^{N}\left(z^{T}(z)\right) - E_{y}\left[\max\left(S^{R}\left(y,z\right),0\right)\right]\right\} > 0,$$

and $y^{R'}(z) = y^{P'}(z) < 0$ if the condition $\beta \lambda^P f G' < 1$ holds. Hence $\partial S^T / \partial z > 0$.

Proof of Lemma 2. From equation (41), we calculate

$$y^{T\prime}(z) = -\Phi_T(z) + \beta \left[1 - \lambda^T \alpha^w \left(1 - G \left(z^T(z) \right) \right) \right] \left(1 - F \left(y^R \right) \right) y^{R\prime}(z) \,.$$

Given $y^{P'} = y_0^{P'} = y^{R'}$, to have $y^{T'}(z) > y_0^{P'}(z)$, it must be true that

(46)
$$\frac{-\Phi_{T}(z)}{1-\beta \left[1-\lambda^{T} \alpha^{w} \left(1-G\left(z^{T}(z)\right)\right)\right] \left(1-F\left(y^{R}\right)\right)} > y^{P'}(z) = \frac{-\Phi_{P}(z)}{1-\beta \left[1-\lambda^{P} \alpha^{w} \left(1-G\left(z^{P}(z)\right)\right)\right] \left(1-F\left(y^{P}\right)\right)}.$$

If $\phi \to 1$ and $\lambda^P \to 0$, then $\Phi_T = \Phi_P = 1$. Because $\lambda^P < \lambda^T$, inequality (46) holds.

Proof of Proposition 2. From the proof of Proposition 1, it is easy to see that

$$E_{y}\left[\max\left(S_{0}^{p}\left(y,z\right),0\right)\right] \geq E_{y}\left[\max\left(S^{T}\left(y,z\right),0\right)\right]$$

if and only if $y_0^P(z) \le y^T(z)$. In Lemma 2, we have established that $d(y_0^P - y^T)/dz < 0$. Hence it is sufficient to show that $y_0^P(z_{min}) > y^T(z_{min})$ and $y_0^P(z_{max}) < y^T(z_{max})$ so that a unique cutoff \bar{z} exists where $y_0^P(z) \le y^T(z)$ for any $z \ge \bar{z}$, and $y_0^P(z) > y^T(z)$ for any $z < \bar{z}$. Using equations (39) and (41), we get

$$\begin{split} \Delta(z) &\equiv y_0^P(z) - y^T(z) \\ &= \beta \left(1 - \lambda^P \alpha^w \rho^P(z) \right) f - \beta \left(1 - \lambda^P \alpha^w(\theta) \rho^P(z) \right) \int_{y^P(z)}^{y_{max}} (1 - F(y)) \, dy \\ &- \beta \phi \lambda^P \alpha^w \int_{z^P(z)}^{z_{max}} S^N(\tilde{z}) \, dG(\tilde{z}) + \beta \left(1 - \lambda^T \alpha^w \rho^T(z) \right) \int_{y^R(z)}^{y_{max}} [1 - F(y)] \, dy \\ &+ \beta \phi \lambda^T \alpha^w \int_{z^T(z)}^{z_{max}} S^N(\tilde{z}) \, dG(\tilde{z}) \, . \end{split}$$

When $\lambda^P \to 0$,

$$\Delta(z_{min}) > \beta f - \beta \int_{y^{P}(z_{min})}^{y_{max}} (1 - F(y)) \, dy$$

Thus, as long as $f > \int_{y^P(z_{min})}^{y_{max}} (1 - F(y)) dy$, $\Delta(z_{min}) > 0$. When $z \to z_{max}$, z^P and z^T converges to z_{max} as well. Hence $\rho^P \to 0$ and $\rho^T \to 0$. Given these, recall $y^R = y^P + f + c$, we have

(47)
$$\Delta(z_{max}) \to \beta f - \beta \int_{y^{P}(z_{max})}^{y^{P}(z_{max}) + f + c} (1 - F(y)) \, dy$$

The left hand side of (47) is less than zero if and only if $c > \int_{y^{P}(z_{max})}^{y^{R}(z_{max})} F(y) dy$. \Box

C Appendix: Data Sources

In this appendix, we provide further information on data sources used in the main text.

C.1 The Workplace and Employee Survey

We use the employer section of Statistics Canada's Workplace and Employee Survey (WES) to measure job creation and destruction in this paper.¹⁸ The WES is an annual, longitudinal, matched employer-employee survey at the establishment level. The WES has two sections, one on employers, and the other on employees. The target population for employer businesses is all establishments in Canada that have paid employees in March, but excluding establishments in the territories and those in crop and animal production, fishing, hunting and trapping, and public administration. The initial sample in 1999 has 6322 establishments, which were drawn from the Business Register (BR) maintained by Statistics Canada, and has been followed over time. In every odd year after 1999, this initial sample was supplemented with a sample of newborn establishments that were added to the BR since the last supplement. Establishments in the WES report the total numbers of permanent and temporary workers in their work forces.

In the employee section, employees are also asked their terms of employment, but only some employees are surveyed from each establishment. All employees in establishments with less than four employees are surveyed, while in larger establishments a sample of workers is selected. A maximum of 24 employees from each establishment are selected.

In measuring job reallocation rates, we use data in the employer section of WES from 2001 onward and use the most encompassing definition of temporary worker. In the 2001 survey and beyond, establishments are first asked their number of employees in the last pay period of March, where an employee is defined as a worker who received a T-4 slip (a slip for income tax purposes issued to workers by employers). Employers are then asked to split the total number of employees into permanent (those who have no set termination date) and non-permanent (those with a set termination date or a specific period of employment). In addition, employers are also asked to report their use of contractors. We consider both non-permanent and independent contractors as temporary workers.

We follow Cao and Leung (2010) and use only data from establishments that operated in every year during the 2001-2005 period. Thus, the target population is establishments survived from 2001 until 2005. There are 6207 establishments in the 2001 WES sample, and 4146 of them survived to 2005. This suggests a 9.6 per cent geometric average exit rate.

Job creation and destruction rates presented in this paper follow Davis and Haltiwanger (1999). A job is created (destroyed) in a workplace, if the net change in employment over the year in that workplace is positive (negative). The job creation (destruction) rate for a workplace is the number of jobs created (destroyed) over

¹⁸For further information on the survey, see *Guide to the Analysis of the Workplace and Employee Survey* - 2004, Statistics Canada, Catalogue no. 71-221-GIE.

the average number of jobs in the workplace in the current and previous year:

$$c_{jt} = \frac{EMP_{jt} - EMP_{jt-1}}{0.5(EMP_{jt} + EMP_{jt-1})} if EMP_{jt} - EMP_{jt-1} > 0,$$

$$d_{jt} = \frac{|EMP_{jt} - EMP_{jt-1}|}{0.5(EMP_{jt} + EMP_{jt-1})} if EMP_{jt} - EMP_{jt-1} < 0,$$

where EMP_{jt} is number of jobs in workplace *j* at time *t*, c_{jt} is the workplace's job creation rate, and d_{jt} is the workplace's job destruction rate.

C.2 Labor Force Survey

The Labor Force Survey (LFS) is a monthly cross-section survey of employment status, hours worked, and hourly earnings by demographic characteristics in Canada. The survey covers non-institutionalized population 15 years of age or older. The LFS is the only source of information for labor market conditions at the national level.

The LFS has a rotating panel sample design, households remain in the sample for six consecutive months. The total sample therefore consists of six representative sub-samples or panels. Every month a panel is replaced after completing its sixmonth stay in the survey. Outgoing households are replaced by households in the same or a similar area.

Starting 1997, the LFS includes information on employment permanency and hourly earnings. Temporary jobs includes seasonal jobs, term or contract jobs, and casual or other temporary jobs. Our measure of temporary employment includes all categories of temporary jobs. We also use the hourly earnings in the LFS to calculate the wage differential between permanent workers and temporary workers, as the LFS sample is representative.

C.3 Survey of Labor and Income Dynamics

The Survey of Labor and Income Dynamics (SLID) is a longitudinal survey of labor force activities, income and sources of income of Canadian workers and households. The SLID is conducted by Statistics Canada over the period 1993 to 2011. The SLID sample is composed of two panels. Each panel consists of two LFS rotation groups and includes about 17,000 households. A panel is surveyed for a period of six consecutive years. A new panel is introduced every three years, so two panels always overlap. Using the SLID data, Statistics Canada calculated the job-to-job transitions for both the permanent employees and temporary employees.

D Labor Market Institutions in Canada

In this section, we give a brief overview of institutional settings regarding the labor market and employment in Canada. In terms of employment protection, on one hand, Canada ranks in between relatively unregulated countries such as the United States, the United Kingdom or New Zealand and heavily regulated countries such as Italy or Spain. Based on OECD research, Cahuc and Zylberberg (2004) (Table 12.4, p. 736) rank Canada above the US, the UK and New Zealand in the degree of employment protection. This ranking has changed little based on more recent documentation by OECD.¹⁹ On the other hand, the regulation on temporary employment in Canada is the weakest among all OECD countries, see for example, OECD (2020). As a result, the Canadian labor market features a fraction of temporary workers at about 14%, smaller than that of Spain at around 30%, but still sizeable. Mapping the employment protection legislation in Canada

¹⁹Canada ranks the third in 2019 among countries with the least strictness of regulation of individual dismissals. See Table 3.3 in OECD (2020).

to our model encounters two difficulties. The first is that labor legislation varies across provinces and types of workers (federal vs. the rest). The second difficulty is that in Canada, as in virtually all other countries but the United States, termination compensation (or severance pay) takes two forms. First, a monetary compensation set as a multiple of the weekly or daily wage and, second, an advanced notice of termination with the likely output loss for the employer. For simplicity, in our theoretical environment the parameter f encompasses both types of firing costs.

All provinces mandate either a monetary compensation or the advanced notice for all workers except those with a very short tenure (less than 3 months in most provinces, and less than 6 months in new Brunswick, Prince Edward Island and Yukon). In Ontario, in addition to termination notice (or payment), workers with a tenure of 5 years or longer are also entitled to receive monetary severance payments, the amount of which increases with the worker's tenure. As an example, in British Columbia no compensation is required if the employee is dismissed after less than three months on the job but long-tenured workers must be compensated through either advanced notice or severance pay up to a maximum of two months per year worked.²⁰

According to Canada Labour Code, federal employers must inform an employee 2 weeks before an employment termination. In lieu of this notice, employers can pay the employee with 2 weeks regular wages. When an employer terminates 50 or more employees within a short period of time, it is required to notice governmental agencies at least 16 weeks before the termination starts. An employee is not required to give an advance notice if she or he terminates the employment. On severance pay, when an employment is terminated by the employer, the employee receives 2 days regular wages for every complete year of service, but the minimum severance pay for a termination is 5 days regular wages.

One assumption made in our theory is that temporary employment lasts for one period only, after which the contract is rescinded or transformed into a permanent one. In reality, firms have some discretion in renewing fixed-term contracts. However, court decisions have limited this discretion by prohibiting employers from repeatedly renewing temporary contracts. Examples of these decisions include Ceccol v. Ontario Gymnastic Federation, 2001 CanLII 8589, and Monjushko v. Century College Ltd., 2008 BCSC 86 (CanLII). In all these cases, the court ordered that a repeatedly renewed fixed-term contract is considered as a permanent contract entitling the employee to compensation. It is difficult to model the flexibility that firms have in renewing, to some extent, the fixed-term contracts and we assume, for simplicity, that firms can only do so for one period (one year, in the empirical application).

In Canada, the employment insurance (EI) program provides unemployment benefits to unemployed workers and workers who pause their job for other reasons like pregnancy or new-born caring. The EI replacement is 55 percent of the worker's average insurable weekly earnings.²¹ The EI regular benefits are paid for

²⁰See https://www2.gov.bc.ca/gov/content/employment-business/employment-standardsadvice/employment-standards. Legislation for other provinces shows a similar structure: see Employment Standards for legislation in Alberta (https://www.alberta.ca/termination-pay.aspx), or (https://www.cnesst.gouv.qc.ca/en/working-conditions/termination-employment) for legislation in Quebec.

²¹As of 2022, the maximum insurable earnings amount is \$60,300, under which the EI payment

a period ranging from 14 to 45 weeks, depending on the unemployment rate in the region where the worker make claims and on the number of hours of insurable employment accumulated during the last 52 weeks before the starting date of EI claims.

is \$638 per week.

E Robustness to Promotion Cost

In the main text, we set the cost of promoting temporary worker to be 1% of the cost of firing permanent workers. In this appendix, we report how our main findings are sensitive to the value of promotion cost. Table 7 summarizes model moments under four values of promotion cost. At larger promotion costs, share of temporary employment falls, because relatively it now becomes less costly to hire permanent workers. Consequently, wage differential between permanent and temporary workers increases. Job-to-job transition of temporary workers increases too since employers now promote fewer temporary workers. Unemployment rate falls as fewer workers are temporary.

Moment	Model	Data	Moment			
	Moment	Moment	c = 0.01 f	c = 0.005 f	c = 0.05 f	c = 0.1 f
$\frac{n^T}{1-u}$	0.094	0.126	0.094	0.097	0.071	0.056
$\alpha^{w}(\theta)$	0.943	0.919	0.937	0.937	0.936	0.937
$\frac{f}{w^P}$	0.179	0.182	0.182	0.182	0.182	0.182
$\frac{\frac{J}{w^P}}{\frac{b}{w}}$	0.467	0.550	0.475	0.475	0.475	0.475
и	0.140	0.084	0.136	0.136	0.135	0.134
$rac{w^P}{w^T} J - J^P$	1.159	1.329	1.151	1.150	1.167	1.180
$\tilde{J} - J^P$	0.115	0.097	0.116	0.116	0.116	0.116
$J - J^T$	0.278	0.256	0.275	0.271	0.303	0.324
a_1	0.173	0.159	0.174	0.174	0.174	0.174
<i>a</i> ₂	0.127	0.193	0.117	0.116	0.127	0.134

Table 7: Model moments under different values of promotion cost

Next, we let the standard deviation of *y* increase by 7.1%, at different values of promotion cost. In doing so, we do not re-calibrate other parameters of the model. Though, magnitude of impacts of increased uncertainty on the labor market can be different when values of promotion costs change, the direction of impacts does not change with promotion costs. Shown in Table 8, share of temporary employment increases at heightened uncertainty at all chosen levels of promotion cost, so does the unemployment rate. Heightened uncertainty still squeezes wage differential, but by little when promotion cost is sufficiently large relative to the firing cost. This is related to smaller wage differentials as promotion incurs larger costs.

	$\frac{n^T}{1-u}$	$\frac{w^P}{w^T}$	и
Before 2008	12.6	1.33	8.4
After 2008	13.2	1.29	8.5
Ratio	1.05	0.97	1.01
Before 2008	9.39	1.15	13.59
After 2008	10.31	1.14	13.94
Ratio	1.10	0.99	1.03
Before 2008	9.75	1.15	13.59
After 2008	10.81	1.14	13.94
Ratio	1.11	0.99	1.03
Before 2008	7.13	1.17	13.54
After 2008	7.75	1.16	13.90
Ratio	1.09	0.99	1.03
Before 2008	5.56	1.18	13.45
After 2008	6.03	1.18	13.81
Ratio	1.08	1.00	1.03
	After 2008 Ratio Before 2008 After 2008 Ratio Before 2008 After 2008 After 2008 After 2008 Ratio Before 2008 After 2008	Before 2008 12.6 After 2008 13.2 Ratio 1.05 Before 2008 9.39 After 2008 10.31 Ratio 1.01 Before 2008 9.75 After 2008 9.75 After 2008 10.81 Ratio 1.11 Before 2008 7.13 After 2008 7.75 Ratio 1.09 Before 2008 5.56 After 2008 6.03	$\overline{1-u}$ $\overline{w^T}$ Before 200812.61.33After 200813.21.29Ratio1.050.97Before 20089.391.15After 200810.311.14Ratio1.100.99Before 20089.751.15After 200810.811.14Ratio1.110.99Before 20087.131.17After 20087.751.16Ratio1.090.99Before 20085.561.18After 20086.031.18

Table 8: Impacts of heightened uncertainty and promotion cost