Housing, Portfolio Choice and the Macroeconomy

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Abstract

This paper investigates the properties of the wealth distribution and the portfolio composition regarding housing and equity holdings, and their relationship to macroeconomic shocks. To this end, I construct a business cycle model in which agents differ in age, income and wealth. Housing provides shelter services and serves as collateral for loans. The model is consistent with several facts such as the life-cycle pattern of housing-to-wealth ratios, the larger degree of concentration for non-housing wealth, the smaller weight of housing in richer households' portfolios as well as the larger housing-to-wealth ratios in recessions. In addition, the model shows that while relaxing the collateral constraint does not impact the business cycle dynamics for the entire economy, it significantly alters the behavior of residential and business investment for the younger and poorer fraction of the population.

Keywords: Heterogeneity, business cycles, life-cycle

JEL Classification: E21, E32, G11

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1 Introduction

Housing accounts for a substantial fraction of wealth in developed economies. In the US, residential structures, with a nominal value of 11 trillion dollars, account for half of the entire capital stock. Residential investment accounts for a third of total investment and about 9% of output. Moreover, from an individual's point of view a house is usually both an investment good and a durable good that provides a flow of "shelter" services. In the data, renters spend about 20% of their total expenditures on housing.

There exists an extensive literature in macroeconomics that compares the wealth distributions implied by equilibrium models with those observed in the data. Following the tradition of one-sector models, these studies have mostly treated capital as a monolith, effectively subsuming the housing stock into an overall measure of capital. Thus, despite the importance of housing in the US economy, housing investment and the consumption of housing services have been largely absent (with very few exceptions) from studies of the wealth distribution.

Previous studies that do consider wealth distribution properties of real estate holdings include Gruber and Martin (2003) and Díaz and Luengo-Prado (2003).[1] They introduce housing (or durable goods in general) into Aiyagari's (1994) framework to evaluate the effects on the level of precautionary savings as well as the ability of the model to deliver the wealth composition and concentration observed in the data. Both studies feature dynastic agents. Yet, as I argue below, the data show a clear life-cycle pattern in the composition of personal portfolios between housing and financial wealth, making the dynastic framework inappropriate for studying this issue. In the financial economics literature, there are a few examples of studies that focus more on portfolio choice over the life cycle with an emphasis on housing. Among them are Cocco (2005) and Cocco, Gomes and Maenhout (2005). However, this line of work does not focus on the effect asset allocations have on

[1] Another example is Silos (2005). In that study, I construct a life-cycle model with a housing rental market and investigate the wealth distribution properties and the choice of tenure pattern over the life cycle.

macroeconomic fluctuations.

In this paper I construct a business cycle model in which agents differ in age, income and wealth and where housing is explicitly modelled. The purpose is to investigate the model's ability to describe properties of the portfolio composition and the wealth distribution and their relationship to macroeconomic shocks.

Age heterogeneity is desirable in any model that deals with housing investment.[2] One of the most salient features of a representative individual's wealth portfolio is the age-dependent pattern–young people accumulate home equity before they start accumulating financial assets. Thus, early in people's lives, housing-to-wealth ratios are large, declining as people accumulate more non-real estate assets, and increasing slightly at the end due to depletion of financial assets during retirement. This pattern is depicted in Figure 1.

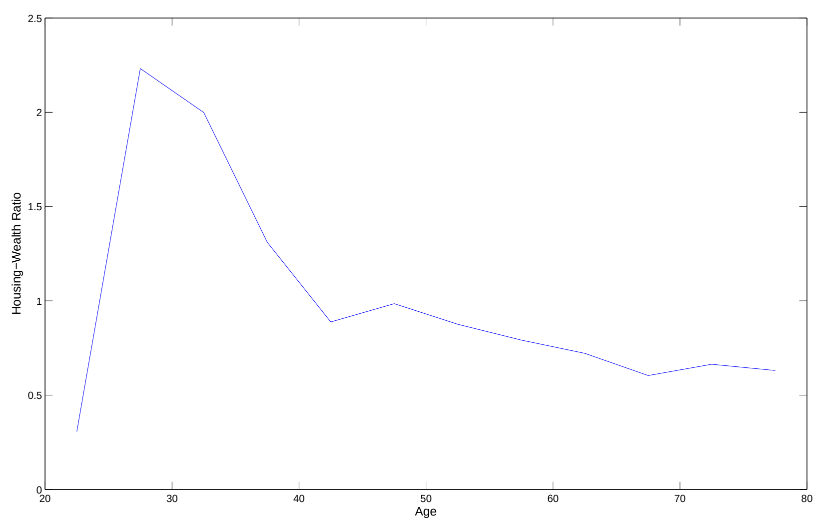

Figure 1: Housing to Wealth Ratio, Source: Survey of Consumer Finances, (2001 and 1998)

To study this feature and others, I construct a standard overlapping generations economy with incomplete markets and macroeconomic shocks, where agents derive utility from

[2]Platania and Schlagenhauf (2000) have an equilibrium life-cycle asset allocation model between housing and equity, modelling the rental market for housing explicitly. The investment is a zero versus fixed house size choice, therefore not valid for studying wealth distribution or even portfolio choice issues.

non-housing consumption, housing services and leisure. Because of heterogeneity across agents, it is the entire distribution of wealth that determines aggregate non-residential capital and labor and hence, interest rates and wages. The solution of the model involves keeping track, at least approximately, of the entire distribution of agents. To do this, I utilize a version of the Krusell-Smith (1998) algorithm in which agents forecast future prices based on a small set of moments of that distribution.

To summarize the main findings, the model does a fairly good job of matching the life-cycle pattern of households' portfolios regarding housing and equity. The housing-to-wealth ratio peaks at ages 26-30, and the slope of the decline matches the data almost exactly. In addition, the model is consistent with housing-to-net-worth ratios being larger for the poor than for the wealthy. As in the data, the housing-to-net-worth ratio is smaller in booms than in recessions, and most of the difference is concentrated in the younger age groups. Regarding the aggregate wealth composition, the fraction of business capital stock in the model is 58%, close to the approximately 50% in the data, and the model delivers a ratio of residential investment to output that coincides with the empirical value.

I analyze the model's business cycle dynamics and more specifically their dependence on the tightness of the borrowing constraints. Aggregate business cycle statistics are similar in economies where agents are allowed to borrow and in those that they are not. In fact, most statistics are similar to what an analogous representative agent model delivers. However, the specification of the borrowing constraint significantly changes the aggregate dynamics for the poorer fraction of the population. In particular the procyclicality of business investment disappears when agents are allowed to borrow.

2 The Model Economy

The model has most features of standard overlapping generations models while introducing elements of real business cycle theory.

At each point in time there is a continuum of agents with unit mass, belonging to one of I generations. Death is certain at age I; hence, the fraction of agents of age $i \in I$ is equal to $1/I$. Individuals are born with zero wealth, work for T years and any given year they have one unit of time to be allocated between working time and leisure.

2.1 Preferences

Individuals maximize their expected lifetime utility over non-housing consumption (c), housing services (s), and leisure (l).

$$U(c, h, l) = E \sum_{i=1}^{I} \beta^{i-1} u(c_i, h_i, l_i) \tag{1}$$

The function u is increasing, continuous, and strictly concave. The notation is standard. The discount factor is denoted by β. Given that death is certain, the effective discount factor is constant across generations.

2.2 Technology

The environment is characterized by uncertain productivity in the aggregate and at the individual level. Agents have different realizations of productivity shocks (denoted by ξ) to their own labor supply, but an aggregate shock z also affects the entire economy, implying different wage and rental rates in each time period. The aggregate technology used to produce output Y combines capital K and labor N through the function $zF(K, N)$, which satisfies the standard properties: strictly increasing, strictly concave, and homogeneous of degree one. Denote the probability of transiting from state z, ξ to state z', ξ' by $\pi(z', \xi'|z, \xi)$.

The resulting output can be either consumed, invested in business capital (k) or invested in residential capital (h). Using this residential capital, the technology for producing housing services is simple. One unit of residential stock h gives one unit of services s. Adjustments to the housing stock imply (aggregate) transactions costs given by the function $\Xi(H_{t+1}, H_t)$. Both types of capital depreciate at rates δ_k and δ_h, respectively.

The aggregate resource constraint in time period t is:

$$Y_t = z_t F(K_t, N_t) = C_t + K_{t+1} + H_{t+1} + \Xi(H_{t+1}, H_t) - (1 - \delta_h)H_t - (1 - \delta_k)K_t \qquad (2)$$

Consumption smoothing is carried out by adjusting the levels of the business capital stock and the residential stock. There are no contingent claims markets for hedging idiosyncratic productivity shocks across individuals. The borrowing constraint is specified as a fraction of the holdings of residential capital, thus resembling the use of housing as collateral in the form of home equity lines of credit, that are popular instruments to smooth adverse shocks. A version of the model will set the fraction to be zero, restricting holdings of capital to be positive.

2.3 Timing

Households first observe both the aggregate shock z and the idiosyncratic shock ξ then they make consumption, investment and leisure decisions, and get labor and capital income. Transfers of residential capital between agents are carried out at the end of the period. This will guarantee that the housing services are enjoyed from the amount of residential stock brought into the period.

2.4 Equilibrium

Before defining the recursive competitive equilibrium it is useful to write the agent's problem in recursive form.

The state variables are the individual's holdings of business capital k, of residential capital h, the individual productivity shock ξ, the aggregate shock z and the joint distribution over assets, ages and productivity shocks, Φ.

Let $r(z, \Phi)$ and $w(z, \Phi)$ denote the interest rates and wages, and η, an age-dependent efficiency factor.[3] Notice that the notation makes clear that prices depend on the entire

[3]Due to the separability in preferences between housing services and non-housing consumption, the model without housing adjustment costs and with a slight modification of the borrowing constraint can be solved with only one state variable. This state variable is the amount of resources or "cash-on-hand" at the start of the period, which comprises the capital gains, the undepreciated business capital, and the undepreciated housing capital.

distribution of agents, since this distribution will determine the aggregate capital-labor ratio the following period. This ratio determines factor prices. The Bellman equation for an individual of age $i \in I$ who is a worker is:

$$V_i(k, h, \xi, z, \Phi) = \max_{\{k', h', l, c\}} u(c, s, l) + \beta \sum_{\xi', z'} \pi(z', \xi'|z, \xi) V_{i+1}(k', h', \xi', z', \Phi'), \tag{3}$$

s.t.

$$c + k' + h' + \Omega(h', h) \leq (1 - l)w(z, \Phi)\xi\eta_i + (1 + r(z, \Phi) - \delta_k)k + (1 - \delta_h)h, \tag{4}$$

$$k' \geq -(1 - \gamma)h, \quad s = h, \quad c > 0, \quad h' > 0, \quad 0 < l \leq 1, \tag{5}$$

$$\Phi' = G(z, \Phi). \tag{6}$$

Agents maximize expected lifetime utility by choosing the levels of housing and business capital holdings, hours worked and non-housing consumption. Equation (4) is the budget constraint for the agent in which the sources of income are capital interest and compensation for labor. In addition to consumption and investment part of income is devoted to pay the costs of adjusting the housing stock, represented by the function $\Omega(h', h)$. The first component of equation (5) is the borrowing constraint which specifies that the agent cannot borrow more than a fraction $1 - \gamma$ of the house she already owns. Equation (6) is the law of motion for the aggregate wealth distribution which agents take as given.

A recursive competitive equilibrium for this economy is a sequence of age-dependent value functions $\{V_i\}_{i=1}^{I}$, policy functions $\{k_i', c_i, h_i', l_i\}_{i=1}^{I}$, factor prices w and r and an aggregate law of motion G such that:

1. The policy functions are the solution to the recursive problem defined by equations (3)-(6) and the value functions satisfy the Bellman equation (3) for an agent of generation i.

2. Factor prices satisfy:

$$r(z, \Phi) = F_K(K(z, \Phi), N(z, \Phi)), \tag{7}$$

$$w(z,\Phi) = F_N(K(z,\Phi), N(z,\Phi)). \tag{8}$$

3. Markets clear:

$$K(G(z,\Phi)) = \sum_{i=1}^{I} \frac{1}{I} \int k_i'(k,h,\xi,z,\Phi) d\Phi \ \ (Asset\ Market), \tag{9}$$

$$N(z,\Phi) = \sum_{i=1}^{I} \frac{\eta_i}{I} \int (1 - l_i(k,h,\xi,z,\Phi)) \xi d\Phi \ \ (Labor\ Market), \tag{10}$$

$$\sum_{i=1}^{I} \frac{1}{I} \int (c_i(k,h,\xi,z,\Phi) + k_i'(k,h,\xi,z,\Phi) + h_i'(k,h,\xi,z,\Phi) + \Omega(h_i'(k,h,\xi,z,\Phi), h)) d\Phi =$$
$$= F(K(z,\Phi), N(z,\Phi)) + (1-\delta_k) K(z,\Phi) + (1-\delta_h) H(z,\Phi) \ \ (Goods\ Market). \tag{11}$$

4. The aggregate law of motion G is generated by the aggregate shock z and the decision rules k' and h'.

The equilibrium defined above can not be computed without some modification. In this model, agents solve a slightly different problem than the one postulated in equations (3)-(6). The reason is their inability to keep track of all the state variables specified, in particular, the income-wealth distribution which is an infinite-dimensional object. I follow Krusell and Smith (1997, 1998) and use a "partial information" approach that replaces the entire distribution with a finite number of moments. In other words, agents keep track of some moments of the distribution and use these to forecast future prices, instead of the entire distribution itself. The final goal of agents is to forecast capital and labor, as they suffice to forecast interest rates and wages. One hopes that these forecasts are accurate enough so as to have negligible quantitative implications when using this computational approach. In Krusell and Smith's work, agents use first-order autoregressive processes (conditional on the current aggregate state) in the logarithm of K (the mean of the distribution) as their forecasting instrument. This turns out to work remarkably well, with very small forecasting errors. Let me rewrite this new "approximate" agent's problem, in which I write the actual laws of motion that agents use:

$$V_i(k,h,\xi,z,K) = \max_{\{k',h',l,c\}} u(c,s,l) + \beta \sum_{\xi',z'} \pi(z',\xi'|z,\xi) V_{i+1}(k',h',\xi',z',K'), \tag{12}$$

s.t.

$$c + k' + h' + \Omega(h',h) \leq (1-l)w(z,K)\xi\eta_i + (1 + r(z,K) - \delta_k)k + (1-\delta_h)h, \tag{13}$$

$$k' \geq -(1-\gamma)h, \quad s = h, \quad c > 0, \quad h' > 0, \quad 0 < l \leq 1, \tag{14}$$

$$lnK' = a_k(z) + b_k(z) lnK \quad z = b, g, \tag{15}$$

$$lnN = a_n(z) + b_n(z) lnK \quad z = b, g. \tag{16}$$

A Technical Appendix at the end provides detailed information on the computations involved in order to solve this problem. In a few words, given values for $a(z) = \{a_k(z), a_n(z)\}$ and $b(z) = \{b_k(z), b_n(z)\}$, agents compute optimal policies. Simulating the economy for a large number of time periods (and for a large number of agents) provides a time series for aggregate capital and aggregate labor. New values for parameters $a(z)$ and $b(z)$ are estimated by least squares. This procedure is repeated as many times as necessary until the values for $a(z)$ and $b(z)$ used by agents roughly coincide with the ones obtained from the aggregate time series. If the forecasting model is adequate (as measured by some metric such as the RMSE, correlation between predicted and actual values, etc.) an equilibrium has been found. If not, it is necessary to either increase the number of moments used or change the functional form in (15)-(16).

3 Parameterization

Given the large amount of time involved in solving the model, I have tried to keep the number of moments that, by construction, match features of US data at a minimum. Whenever possible, I have assigned parameter values drawing on previous sources or directly estimating empirical counterparts from the data.

3.1 Demographics

Agents live for $I = 60$ periods. I will assume that the first age in the model corresponds to 21 years old and they die with certainty when they reach 80. Individuals work for $T = 40$ periods, thus retiring when they are roughly 60 years old. The absence of a Social Security scheme implies that when retired, agents live off wealth accumulated during the working years.

3.2 Preferences and Endowments

The utility function chosen is of the logarithmic class, commonplace in traditional business cycle models:

$$u(c, h, l) = \theta ln(c) + (1 - \theta)ln(s) + \omega ln(l) \tag{17}$$

To obtain an estimate of ω, one of the moments targeted to match a feature of the US data is an average of 32% of time devoted to work. Notice that this is an average over the entire population, including retirees: agents of working-age devote a larger fraction. This average determined a value for ω of roughly 1.05. This value was used for all versions of the model, the aggregate average was a target only in the first version, which allows no borrowing. The discount factor was set at 0.96, which is the value used by Gourinchas (2000), but smaller than values used in other life-cycle models. However, the absence of death uncertainty in this model makes it difficult to compare discount factors across these different studies. The value for θ, the share of non-housing consumption in the utility function, is set at 0.8. This value has been used by Peterson (2003) and it is consistent with the share of housing expenditures being roughly 20% in the Consumer Expenditure Survey.[4]

The labor endowment process is a finite-state approximation of the model for the

[4]Note: We can only measure expenditures, while the utility function is defined over service flows, which in the case of housing can be very different. This 20% figure is a rough approximation.

idiosyncratic component of labor earnings estimated in Storesletten, Telmer, and Yaron (2004). Their sample is annual covering the period 1968-1993, with data from the Panel Study of Income Dynamics (PSID). Denoting by $u_{it} = ln(y_{it})$ the logarithm of the idiosyncratic component of labor income for household i at time t, the model estimated is:

$$u_{it} = z_{it} + \epsilon_{it} \tag{18}$$

$$z_{it} = \rho z_{i,t-1} + \nu_{it}$$

where $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$ and $\nu_{it} \sim N(0, \sigma_\nu^2)$. Peterson (2003) and Fernández-Villaverde and Krueger (2002) report $\rho = 0.935$, $\sigma_\epsilon^2 = 0.017$ and $\sigma_\nu^2 = 0.061$. I have approximated this process as a three-state Markov Chain,[5] normalizing the average value for the idiosyncratic shock to be 1. The resulting support for $y = e^u$ is the set $\{0.628, 0.946, 1.426\}$ with transition probability matrix:

$$\Omega = \begin{bmatrix} 0.854 & 0.146 & 0.000 \\ 0.105 & 0.790 & 0.105 \\ 0.000 & 0.146 & 0.854 \end{bmatrix} \tag{19}$$

At any time period the proportions of agents with high, middle, and low productivity are $0.295, 0.410, 0.295$, respectively. The goal of Storesletten *et al.*'s paper is to estimate a model with counter-cyclical volatility to show that in recessions the uncertainty about future earnings is larger. The model in this paper is suitable for analyzing implications for business cycle analysis of such a process. This can be particularly relevant for the behavior of residential investment, as shown in Peterson (2003). For simplicity I have restricted the income process to have a constant variance.

In addition to this idiosyncratic productivity shock, agents face an age-dependent

[5] The reason to have three states is that it is the minimum dimension with which I can introduce some degree of skewness. In some cases though, and probably more so here given that the original continuous state-space process is an ARMA(1,1), it is desirable to trade off some accuracy on the dimension of the state space to enrich the dynamics of the underlying autoregressive process. Making the dynamics richer while keeping the three states was computationally infeasible. See Silos (2006) for an extensive discussion and approach to evaluating that tradeoff.

efficiency profile $\{\eta_i\}_{i=1}^I$ used in Huggett and Ventura (1999) [6]. Hansen (1993) estimated median wage rates from the Current Population Survey (CPS) for different age groups. Huggett and Ventura used Hansen's estimates and set them to be the wage corresponding to the age in the center of the group and linearly interpolated to obtain values for all ages.

3.3 Technology

The functional form chosen for the aggregate production function is a Cobb-Douglas on labor and capital, $F(K_t, N_t) = K_t^\alpha N_t^{1-\alpha}$, with $\alpha = 0.36$. This number implies a labor share in national income of about 0.64, and that value has been used previous studies such as Fernández-Villaverde and Krueger (2002).

The depreciation rates are obtained directly from the data in the following way. In a non-stochastic steady state, the capital accumulation equations in the model imply the following relationship between output, investment and the depreciation rates:

$$\frac{I_k/GDP}{K/GDP} = \delta_k \tag{20}$$

$$\frac{I_h/GDP}{H/GDP} = \delta_h \tag{21}$$

In US annual postwar data[7] these two expressions imply values of $\delta_k = 0.094$ and $\delta_h = 0.043$.

The adjustment costs function Ω is specified to be quadratic following the literature on portfolio choice (e.g., Heaton and Lucas (1996)):

$$\Omega(h', h) = \frac{\zeta(h' - h)^2}{2}$$

The value of ζ was chosen so that the contemporaneous correlation between residential investment and output was positive in all economies considered. This in turn implied a value of ζ of 0.1.

The number of states for the aggregate productivity shock is set at two, a recession

[6] Mark Huggett kindly provided these data.
[7] The reader is referred to the appendix for definitions of data variables used throughout the paper.

value and an expansion value. In a manner similar to Prescott (1986), I have estimated the parameters ρ and σ in an autoregression of the deviations around a linear trend of the Solow residual. The frequency is quarterly and the sample is 1964-2003.

$$ln\, z_t = \rho\; ln\, z_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2) \tag{22}$$

The parameter estimates were $\rho = 0.9329$ and $\sigma = 0.0073$.[8] In the yearly frequency these imply an unconditional standard deviation of 2.05% and a first-order autocorrelation of 0.759 for z_t. I have approximated this process as a two state Markov chain that matches these two moments. The support for z is the set $\{0.9795, 1.0205\}$ and the transition probability matrix is

$$\Pi = \begin{bmatrix} 0.879 & 0.121 \\ 0.121 & 0.879 \end{bmatrix} \tag{23}$$

Finally, I need to specify a value for γ, where $1-\gamma$ is the maximum fraction of the house against which agents can borrow. Notice that the constraint involves h, the house currently owned, rather than h', the house the agent wants to buy. This bears closer resemblance to a home equity line of credit (HELOC) than to a mortgage. A HELOC is a loan that uses the house currently owned as collateral for whatever expenses the agent needs to finance. The nature of the constraint allows me not to have to take a position regarding the ultimate use of the amount borrowed, whether it is to buy more residential stock or non-housing consumption goods. The empirical counterpart that most closely resembles $1-\gamma$ would be the maximum fraction of the house that banks allow homeowners to borrow against in the form of a HELOC. Private conversations with commercial banks reported values for $1-\gamma$ of about 0.9, and the range was roughly 0.8-1.[9] These values are similar to assigned minimum downpayment fractions in the literature when h' is used

[8] Prescott (1986) reports $\rho = 0.95$ and $\sigma = 0.00763$. My estimates reflect the smaller volatility of movements of GDP around trend in the United States during the '90s.

[9] These fractions apply to the market value of the house net of mortgages, i.e., they apply to the value of the "part" of the house the individual actually owns. At this point, the model is too simple to account for this difference.

in place of h in the borrowing constraint. See for example Fernández-Villaverde and Krueger (2002), Díaz and Luengo-Prado (2003) or Peterson (2003). In Sections 3 and 4 I will present results with two values of $1-\gamma$: 0 (no borrowing) and 0.9. [10]

Table 1 provides a summary of parameter values and their target/source.

Table 1: Summary of Parameter Values

Parameter / Variable	Value	Target / Source
β	0.96	Gourinchas (2000)
ω	1.2	1/3 time at work
θ	0.8	20% exp. in housing; Peterson (2003)
ξ	—	Storesletten, Telmer, and Yaron (2004)
η	—	Huggett and Ventura (1999)
α	0.36	NIPA
δ_k	0.094	”
δ_h	0.043	”
z	{0.9795,1.0205}	Solow residual; NIPA
$1-\gamma$	0.9	Commercial Banks
ζ	0.1	$Corr(GDP_t, I_{h,t}) > 0$

4 Results

4.1 Aggregate Wealth Composition and the Business Cycle Facts

The first column in Table 2 presents a few selected aggregate ratios for the United States economy during the period 1964-2003. All quantities were computed at an annual frequency. The most noticeable feature is that the total capital-output ratio seems somewhat larger than values previously reported in other studies, such as Cooley and Prescott (1995). However, my definition of GDP does not include housing services, which are about 9%-10% of total output in the data. In the model there are implicitly two production

[10] I computed results with $1-\gamma$ equal to 0.8 and 1. Results are quantitatively similar to $1-\gamma$ equal to 0.9 and therefore are not reported in the paper. However, they are available upon request.

functions: one given by the Cobb-Douglas technology and another given simply by $s = h$. Both provide "flows" to the consumer, but what I call GDP in the model is only the Cobb-Douglas part. Analogously, the definition of non-housing consumption includes all services, except the ones derived from housing (it also excludes expenditures in consumer durables).

Table 3 describes some familiar business cycle facts for a few key variables: output, consumption, investment and employment. The data were logged and detrended using a Hodrick-Prescott filter, and results are shown for deviations of variables around that trend. The smoothing parameter in the HP filter was set at a value of 100.[11]

Table 2: US data (1964-2003) vs. Model Economies

Variable	US data	$1-\gamma=0$	$1-\gamma=0.9$
$(I_k+I_h)/GDP$	0.260	0.340	0.329
I_h/GDP	0.081	0.076	0.079
I_k/GDP	0.179	0.264	0.250
C/GDP	0.740	0.660	0.671
$(K+H)/GDP$	3.471	4.410	4.338
K/GDP	1.754	2.637	2.505
H/GDP	1.717	1.772	1.833
$K/(K+H)$	0.505	0.598	0.577
$H/(K+H)$	0.495	0.402	0.423

The business cycle stylized facts are apparent from the table. Investment, consumption and employment are all highly procyclical, with all contemporaneous correlations with output being larger than 0.7. Another familiar fact is the high volatility of investment relative to output and consumption, and the smaller variance of the former relative to the latter.

[11] There is no unanimity among economists regarding the choice of the smoothing parameter when dealing with yearly data. Results in the literature boil down to choosing n and setting $\lambda_{year} = 1600*0.25^n$. The values chosen for n range from n=1, $\lambda_{year} = 400$ (e.g., Cooley and Ohanian (1991)) to $n = 3.8$ (e.g., Ravn and Uhlig (2002)). The choice of $n = 2$, $\lambda_{year} = 100$ is an intermediate value used in Rios-Rull (1996) and in Backus and Kehoe (1992).

Table 3: US data (1964-2003)

	Std. Dev.		Corr(GDP,x_t)
	Absolute	Rel. to GDP	
GDP	0.027	1.000	1.000
Consumption	0.012	0.431	0.778
Aggregate Investment	0.083	3.015	0.962
Business	0.066	2.412	0.747
Residential	0.186	6.796	0.851
Employment	0.005	0.172	0.788

4.1.1 Tight Borrowing Constraint

This section presents results for the economy in which $1-\gamma$ is set to zero. The second column of Table 2 shows the steady state values for several ratios for the no-borrowing economy. Throughout the paper, all moments are computed as averages obtained from a long time series simulation, as opposed to a sequence of shorter series. The figures are roughly consistent with US long-run averages, and in some aspects the model performs remarkably well: the ratio of residential investment to GDP roughly coincides with the empirical value, and the fraction of residential capital to output is very close (1.7 vs. 1.8).

On the contrary, the total capital to output ratio ($\frac{K+H}{Y}$) of 4.410 seems large compared to the value reported in the first column, 3.471. It is expected that the introduction of borrowing in the economy should partly alleviate this problem. The presence of a very tight borrowing constraint increases the agents' desire of saving in an "anticipatory" sense, to avoid being constrained in the future. The magnitude of this effect will be discussed in the next section.

Table 4: Tight Borrowing Constraint, ($1-\gamma=0$)

	Std. Dev.		Corr(GDP,x_t)
	Absolute	Rel. to GDP	
GDP	0.018	1.000	1.000
Consumption	0.007	0.388	0.970
Aggregate Investment	0.040	2.219	0.996
Business	0.053	2.904	0.857
Residential	0.093	5.141	0.244
Employment	0.005	0.290	0.977

Another possible explanation is the absence of a Social Security scheme that would help finance retirees' expenditures. As Huggett (1996) notes, models without Social Security do a poorer job in matching capital-output ratios because of the overaccumulation of capital undertaken by individuals during their working years.

The large capital-output ratio translates into a small value for the return on equity. In this economy it is 3.65%. In the data, the relevant return to be mapped with the model is the return to the entire business capital stock. This is difficult to measure accurately, but back-of-the-envelope calculations suggest that it is of the order of 11% for the sample considered.[12] This might imply that the discount factor used in the model is rather large.

Regarding business cycle statistics, Table 4 reports standard deviations and correlations with output for the same variables shown in Table 3 for the economy for a tight borrowing constraint. The model output was, as the data, logged and detrended with an HP filter.

The economy with a tight borrowing constraint is roughly consistent with the business cycle facts, particularly when only focusing on the main four aggregates: output, consumption, investment, and employment. Standard deviations have the correct magnitudes, and the volatility of employment almost matches the data exactly. Consumption,

[12]The way I have arrived at this figure is by using the relationship $r=\frac{\alpha GDP}{K}$ and subtracting the depreciation rate δ_k. Taxes have been ignored and, hence, this number should be taken as a rough approximation.

employment, and investment are all procyclical. The proportion of the variance of output in the data that is explained by technology shocks is 75%, similar to findings in Cooley and Prescott (1995), who report 62%.

The model has difficulty delivering the high contemporaneous correlation between output and residential investment. Although results are not presented, portfolio adjustment costs are key in delivering the (weak) procyclicality of residential investment. Absent these costs the contemporaneous correlation is negative, about -0.2. The magnitude of volatilities of both types of investment relative to output are close to their empirical counterparts.

4.1.2 Loose Borrowing Constraint

The model in this section replaces the constraint $k' \geq 0$ by $k' \geq -(1-\gamma)h$ with $1-\gamma = 0.9$. The last column of Table 2 reports long-run averages of several aggregate ratios for the economy with a loose borrowing constraint.

Results are very similar to the ones shown in the second column of that same table, i.e to the economy with tight borrowing constraints. The model still matches exactly the ratio of residential investment to GDP, the fractions of business and residential capitals in the economy have barely moved. The introduction of the ability to borrow in the economy brought the values of the total capital-to-output ratios closer to the data, but they are still considerably large.

Table 5: Loose Borrowing Constraint, ($1-\gamma=0.9$)

	Std. Dev.		Corr(GDP,x_t)
	Absolute	Rel. to GDP	
GDP	0.019	1.000	1.000
Consumption	0.008	0.426	0.928
Aggregate Investment	0.043	2.258	0.989
Business	0.065	3.430	0.806
Residential	0.133	6.997	0.075
Employment	0.006	0.329	970

Table 5 reports the business cycle statistics for the economy where borrowing is allowed. The correlation pattern of macroeconomic variables with output is very similar to the ones reported for the tight constraint case. Again, all variables are procyclical, residential investment is not as procyclical as in the data. Its correlation with output is now somewhat smaller than in the economy with a tight constraint (0.08 on average versus 0.24), drifting its value away from what the data say. Regarding the standard deviations, they remain roughly unchanged for output, consumption and employment, but slightly increase for investment. In particular, the standard deviation of residential investment relative to output increases from approximately 5.1 to about 7, which is the value observed empirically.

Although the aggregate business cycle statistics look similar for the two borrowing constraints specified, they are quite different when the focus is on agents who are close to or at the borrowing constraint. In particular, I have focused on agents of the first six generations and have analyzed the business cycle behavior of this group independently. The two panels in Table 6 show how the investment and consumption patterns of the younger generations depend on the borrowing constraint specified.

Table 6a: Business Cycle Dynamics – Young Agents – $1 - \gamma = 0$

	Standard Deviation	Correlation with GDP
Consumption	0.009	0.904
Business Investment	0.173	0.774
Residential Investment	0.031	0.929

Table 6b: Business Cycle Dynamics – Young Agents–$1 - \gamma = 0.9$

	Standard Deviation	Correlation with GDP
Consumption	0.008	0.843
Business Investment	0.029	-0.179
Residential Investment	0.031	0.658

The most noticeable feature is the weaker cyclicality of business investment with the loose borrowing constraint. Young agents accumulate residential capital faster than mid-age agents to quickly acquire greater borrowing ability. Debt levels for the first age groups will get larger, even when the economy is in an expansion, causing the weaker correlation between output and business investment. The other side of the coin is the larger procyclicality of residential investment relative to the aggregate economy: correlations with output increase from 0.24 to 0.93 in the economy with a tight constraint and from 0.07 to 0.66 in the case of a loose constraint.

4.2 Portfolio Choice over the Life Cycle and the Business Cycle

This section examines the cross-sectional patterns of portfolio composition in all versions of the model. Of special interest is trying to match the life-cycle pattern of real estate and equity holdings.

4.2.1 Data

The Survey of Consumer Finances (SCF) has become the main source used by financial economists to address any question related to the composition of balance sheets in US households. I have used the 1998 and 2001 versions in which roughly 4,400 families were interviewed [13]. The SCF gives great detail on the housing side of households' asset position. It provides responses about quantities owed from different mortgages, HELOCs, market values of primary residence, values of vacation homes, and other real estate participations. Additionally, it gives information about types of mortgages (e.g., whether it is a mortgage from the Veterans Administration, the Federal Housing Administration, etc.), frequency of payments, real estate taxes, number of units in the lot, etc.

For the purpose of mapping data and model, it is important to realize that an explicit modelling of a rental market is absent from this paper. Everybody is a homeowner. To be consistent, I eliminated renters from the sample, individuals that have zero housing wealth. There is still some ambiguity, however, on which variable from the SCF best provides the best match to the variable h in the model. In general individuals enjoy an entire home, but spend a large fraction of their lives paying for it. This implies that, in general, agents do not own the house they live in, at least entirely. In the 2001 SCF, 35% of homeowners reported owning the entire value of their primary residence. In the model, even more so in the tight constraint case, these two concepts can not be easily differentiated. The two candidate variables for h are the value of the primary residence and home equity. Home equity is defined as the value of the primary residence minus the sum of all outstanding mortgages minus the sum of all loans using the house as collateral (in the form of HELOCs). Agents in the model demand housing based on the marginal utility of housing services. In addition, the definition of home equity implies the existence of debt that can only be used to finance a home purchase. For these reasons, the variable that describes the housing position when comparing data and model is the value of the primary residence. This is consistent with other work; for example Diaz and Luengo-Prado (2003). The SCF variable that corresponds to total wealth is "Net Worth", which is defined as total assets minus total debt.

[13] For a detailed description of the variables used please refer to the Technical Appendix.

4.2.2 Portfolio Choice Over the Life Cycle

Figure 1 shows the life-cycle profile of the ratio of housing to total wealth.[14] Young agents have little financial wealth, very often negative, and the value of the home in which they live exceeds their total wealth by a factor between two and three. They borrow because they expect their income to increase due to deterministic increases in productivity as agents get older. During their working years agents accumulate financial assets, hence the housing to wealth ratio decreases, increasing mildly at the end of their lifetime when non-housing assets are used to finance retirement.

Figure 2 compares the output from both models with the data. It is clear that the performance of the economy with a tight borrowing constraint is very poor. The housing-to-wealth ratio barely varies over the life cycle while empirically we observe a large drop over the life cycle of an agent. The agents' inability to borrow bounds the housing-to-wealth ratio to 1. Large housing-to-wealth ratios are the result of the steepness of housing accumulation over the initial periods of an agent's life at the expense of issuing debt to be repaid later in life. In short, a model where agents are not allowed to borrow does not seem to replicate basic features of the data.

The model in which agents are allowed to borrow can accurately capture the rate of decline in the importance of housing in households' portfolios as the agents age. When $1-\gamma$ equals 0.9, the peak in the housing-to-wealth ratio occurs in the second age group (26-30), consistent with the empirical evidence, although the peak value is somewhat larger than what is observed empirically. The rate of decline in the housing-to-wealth ratio that occurs during the life cycle until retirement coincides with its empirical counterpart.

An important thing to notice is that the aggregate housing to wealth ratio, as I have measured it in the Survey of Consumer Finances, does not match with the aggregate housing-to-total-capital ratio calculated from the National Accounts. This can be due to several reasons such as measurement error, small sample size, etc. Given the variety

[14] I have computed averages based on 5-year age groups.

of causes, it is difficult to take into account this discrepancy in a reasonable way. An implication of this is that the model understates the magnitude of the housing-to-wealth ratio, except in the initial and final years of the individual's life.

Figure 3 shows the ratio of housing to total net worth by level of wealth. I have computed the ratio for two different groups. The first includes the wealthiest 5% of individuals in the sample. The second is the rest of the population (bottom 95%). With few age group exceptions, the ratio of housing-to-wealth ratio increases as wealth decreases. For the richer fraction the pattern of the portfolio allocation is completely different than that of the poorer agents. Although it is larger at the beginning of the agent's life, the difference is small: the mean housing-to-wealth ratio is about 0.17 for the younger age groups, and approximately 0.05 for older agents. On average, wealthy individuals have an almost 15 times smaller housing to wealth ratio than the poorer fraction of the population. Figure 4 shows the model's implication for the portfolios of the rich and the poor. When disaggregated by wealth, the difference in the model occurs only in the younger age groups, as opposed to over the entire life cycle, as it is observed in the data. This is caused by the small amount of borrowing that occurs at mid-ages. Since for those agents borrowing constraints are not binding, preferences are homothetic for both the rich and the poor and we should not expect differences in their housing-to-wealth ratios. Finally, the magnitude of the difference between the importance of the home in households' portfolios when sorted by wealth is much smaller in the model than in the data. For the parameterization considered, the maximum difference is 0.89.

4.2.3 Portfolio Choice Over the Business Cycle

Regarding the allocation between real estate and equity over the business cycle, the most characteristic feature is the larger weight of the home in households' portfolios in recessions rather than in booms. The life-cycle portfolio pattern in booms and in recessions is shown in Figure 5.[15] It is noticeable though, that the difference is concentrated in the initial age

[15]Due to the small number of years for which the Survey of Consumer Finances is available, there are no other recession years besides 2001. In National Accounts, however, the correlation between GDP

groups with no discernible pattern in recessions versus expansions for older agents. As seen in Figures 6, the model economy is again able to reproduce these facts: the weight of the home is reduced in booms and it only occurs for the younger agents. In the model, the reason aggregate shocks affect mostly the younger generations is that borrowing-constrained agents are concentrated mainly in those groups. Borrowing constraints imply non-homotheticity, therefore changing the ratio of assets when agents face an income shock.

4.2.4 Wealth Inequality

Finally, Table 7 reports values for the Gini coefficients by type of asset. In US data, the Gini index for non-housing wealth, i.e., net worth minus home equity, is 0.67 and it is smaller for housing wealth (0.59). The pattern of concentration levels over the life also differ across types of assets as is apparent in Figure 7. It depicts the Gini indices for different age groups for housing wealth and non-housing wealth. For non-housing wealth, at younger ages the wealth distribution is more unequal given the existence of many agents that are net borrowers. As agents age, the very small proportion of individuals with negative wealth decreases the Gini indices. For housing wealth, not only is the level of concentration smaller for all age groups, but the levels vary less across age groups relative to non-housing wealth. The model economies all deliver the qualitative fact that concentration is larger for non-housing wealth than for housing wealth. However, quantitatively, there is a large difference in magnitude for the Gini indices in housing wealth. The models deliver coefficients that are less than half of what is observed in the data. The maximum Gini for housing wealth is 0.231. Model economies perform much better delivering the correct magnitude for the degree of concentration in non-housing wealth. Gini coefficients are about 0.75 for the economy with a loose borrowing constraint, and even for the tight borrowing constrained economy the Gini index is as high as 0.58.

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growth and the ratio of the residential stock to the total capital stock ($\frac{H}{K+H}$) is -0.25.

Table 7: Gini Coefficients by Type of Wealth

Economy	Non-Housing Wealth	Housing Wealth
US Data	0.676	0.590
$1-\gamma=0$	0.586	0.231
$1-\gamma=0.9$	0.763	0.227

5 Conclusion

The goal of this paper was to assess the ability of a standard macroeconomic model to describe features of the wealth distribution and portfolio composition observed in US data regarding housing and financial wealth. Despite its simplicity, the model can account jointly for wealth distribution moments as well as facts regarding economic fluctuations. More specifically, the model can deliver the typical life-cycle pattern of housing-to-wealth ratio with a peak at young ages and a decrease throughout the agent's life. The peak occurs at the age group 26-30 with a value between 2 and 3, roughly what is observed in US data. Another fact regarding housing and equity holdings is that the home is a relatively more important asset for poorer agents than for the richer. Although the model underestimates the magnitude of this difference, it can qualitatively account for the fact that the housing-to-wealth ratio is smaller for wealthier households.

The relationship between macroeconomic shocks and portfolio choice has also been investigated. According to available data, the housing-to-wealth ratio is larger in recessions than in booms, and most of the difference occurs in young agents' portfolios. The models presented in this paper are also consistent with this fact. I have also analyzed how different specifications of the collateral constraints affect business cycle dynamics. The main result is that allowing for borrowing does not affect greatly the aggregate statistics for the entire economy. However, the cyclicality and volatilities of business and residential investment for the younger and poorer fraction of the population is very sensitive to the tightness of the borrowing constraint.

6 Technical Appendix

6.1 Definition of Data Variables

6.1.1 National Accounting Data

Almost all of the aggregate data comes from the Bureau of Economic Analysis website (*www.bea.gov*). The only exceptions are the United States population, the average weekly hours worked and the number of employees in the private sector, all of which come from the Bureau of Labor Statistics Website (*www.bls.gov*). The data are annual (except when extracting the Solow residual, see below) starting in 1964 and ending in 2003.

- *Gross Domestic Product*: Output is defined as Gross Domestic Product minus Consumption Expenditures in Durable Goods minus Expenditures in Housing Services minus Net Exports minus Government Consumption and Investment Expenditures. To compute the business cycle moments (standard deviations and cross-correlations with output), output was transformed into *per capita* terms through dividing by the US population and transformed into real terms by deflating using the GDP deflator.

- *Investment*: Aggregate investment is Total Gross Private Domestic Investment. Business investment is the sum of non-residential investment in structures, equipment and software. Residential Investment is Total Investment minus Business Investment.

- *Consumption*: Consumption is defined as Personal Expenditures in Consumption minus Expenditures in Durable Goods minus Expenditures in Housing Services. Investment and Consumption were also deflated by the GDP deflator and transformed into *per capita* terms through dividing by the US population.

- *Employment* The definition of employment is the average weekly hours of production workers in the private sector. (code CES0500000005, Bureau of Labor Statistics).

- *Capital Stocks*: The stocks of both residential and business capital come from the Fixed Assets Tables (Current Net-Cost). The definition of Residential Capital is

Residential Structures. Business capital is defined as Total Private Fixed Assets minus Residential Structures.

- *The Solow Residual*: The computation of the technology process amounts to fitting a first order autoregression to the deviations from a linear trend of the logarithm of the Solow Residual. Let $GDP_t = A_t K_t^{\alpha} N_t^{1-\alpha}$. This implies the log-linear relationship $ln(A_t) = ln(GDP_t) - \alpha ln(K_t) - (1-\alpha) ln(N_t)$. The empirical counterparts in this equation are not defined in per capita terms. The definition of employment is average weekly hours (same code as above) times the total numbers of production workers in the private sector (code CES0500000001, BLS). The measure of Gross Domestic Product included net exports but excluded government consumption and investment expenditures. The frequency in this calculation is quarterly. The Bureau of Economic Analysis does not provide quarterly estimates of the capital stock. I constructed the quarterly series by applying the perpetual inventory method, using the investment flows, assuming a depreciation rate of 0.025 per quarter and fixing the initial capital stock to be the 1963 estimate from Table 1.1 in the Fixed Assets section of the Bureau of Economic Analysis National Accounts. Investment was defined as quarterly Gross Private Domestic Investment. Capital Stocks and Gross Domestic product were deflated using the quarterly GDP deflator.

  Once $ln(A_t)$ has been computed, its deviations around a linear trend are the empirical counterpart of $ln(z_t)$ in the model.

6.1.2 Wealth Data

Data on the wealth distribution comes from the 2001 Survey of Consumer Finances (SCF). This survey provides information about the wealth composition, income, and demographic variables. It is sponsored by the Federal Reserve Board and collected by the National Organization for Research at the University of Chicago. It is conducted every three years and its sample size is relatively small, interviewing around 4,500 families.

The SCF oversamples wealthier families, given the high level of concentration of the wealth in the United States, therefore appropriate weights need to be used to compute statistics from this dataset. All calculations reported in this paper are weighted averages.

Definitions of Variables: I have defined variables in the same way as Aizcorbe, Kennickell, and Moore (2003).

- *Financial Assets*: Instruments in this category include checking accounts, savings accounts, money market accounts (including the ones in mutual funds), call accounts at brokerage houses, certificates of deposit, stocks (including stocks at mutual funds), government bonds (including mutual funds), tax free bonds, mortgage-backed bonds, corporate and foreign bonds , IRAs (and other quasi-liquid retirement accounts), account type pension plans (including 401(k)'s), life insurance and other financial assets (including among other things cash or royalties).

- *Non-Financial Assets*: Includes the value of all vehicles, the value of the primary residence and other real estate participations, vacation homes, net equity in business at market value, and other non-financial assets (such as jewelry, art, rare books, etc. . .).

- *Home Equity*: Defined as Value of Primary Residence minus the sum of all mortgages (up to three mortgages) minus the value of all home equity lines of credit (HELOCs) on the primary residence.

- *Debt*: Housing debt (which includes debt on primary residence and all other residential property), credit card debt, other installment loans, loans against pensions, against life insurance, and any other miscellaneous loans.

- *Net Worth*: It is defined as Total Assets (financial and non-financial) minus Total Debt.

In computing the averages reported in the paper I focused solely on homeowners. In addition, responses of an exact zero value for net worth were also eliminated. The reason was simply to avoid dividing by zero when looking at housing to wealth ratios. The treatment of outliers was rather rudimentary: I decided to leave out families that reported housing to net worth ratios larger than 100 (in absolute value). The number of eliminations was not large but some ratios reported were over 1000, which given the small sample had a large impact on the age-group averages.

6.1.3 Income Data

The parameterization of the idiosyncratic earnings process was taken from Peterson (2003) Fernández-Villaverde and Krueger (2002) who use estimation results from Storesletten, Telmer and Yaron (2001) (STY). STY obtained annual data from the Panel Study of Income Dynamics (PSID) from 1968 to 1993, and constructed 24 3-year repeated panels to estimate the earnings model. Earnings are defined as wage earnings by the head of the household plus female wage earnings plus total transfers to the household. The latter include unemployment insurance, transfers by non-household members and workers' compensation. Total earnings are transformed into per member earnings by dividing by family size, and deflated to 1968 dollars using the CPI.

6.2 Computational Details

This section provides a step-by-step description of the model solution.

1. Specify grids $\mathcal{K} = \{k_1, \ldots, k_K\}, \mathcal{H} = \{h_1, \ldots, h_H\}$ and $\mathcal{K} = \{K_1, \ldots, K_N\}$ for individual business capital holdings, individual housing holdings and aggregate capital-labor ratios.

2. Guess an initial value for the parameters $a(z), b(z)$ in (15)-(16).

3. Solve for the optimal decisions at all age groups, levels of income, wealth, and aggregate states (aggregate capital-labor ratio and technology shock). The way these policy functions are computed is by approximating them as piecewise linear functions and solving the nonlinear system resulting from grouping first-order conditions

and the budget constraint. The first order conditions give the following three non-linear equations (after substituting for the Lagrange multiplier, and eliminating individual-level holdings as states to simplify notation):

$$0 = \theta/c_i(K,z,\xi) - \beta\{\sum_{z',\xi'} \pi(z',\xi'|z,\xi)c_{i+1}(K',z',\xi')(1+r(z',K')-\delta_k)\} \quad (24)$$

$$0 = \theta/c_i(K,z,\xi) - \beta\{\sum_{z',\xi'} \pi(z',\xi'|z,\xi)(\frac{1-\theta}{h_i(K',z',\xi')} + (1-\delta_h)\frac{\theta}{c_{i+1}(K',z',\xi')})\} \quad (25)$$

$$0 = \omega - (1-n_i(K,z,\xi))\frac{\theta}{c_i(K,z,\xi)}w(K,z)\eta_i\xi \quad (26)$$

And finally, the budget constraint:

$$0 = c_i(K,z,\xi) - (1-\delta_h)h - (1+r(K,z)-\delta_k)k - w(K,z)n_i\eta_i\xi +$$

$$+k_i(K,z,\xi) + h_i(K,z,\xi) \quad (27)$$

The problem for retirees is simpler: there is no (26) and (27) does not include the wage term. In case of a corner solution a constrained system is solved in which the first Euler equation disappears and the decision rule k_i is set to the lower bound in capital holdings. The goal is to solve for the four decision rules $c_i, h_i, k_i, n_i, \forall i \in I$ in all possible states. Interpolation between grid points is linear in three dimensions (k,h and K). Regarding the non-linear equation solver, in this paper I have used the FORTRAN routine "hybrd1" in MINPACK ©[16].

4. Once all decision rules have been calculated it is necessary to simulate the economy. Starting with an initial distribution over wealth and income, the economy is simulated forward in the following way: at each point in time I add an initial 21-year old generation with a distribution over income that matches the fractions implied by (19) and with zero holdings of business capital and housing capital. With a known

[16] This package was developed by the Argonne National Laboratory, and it is freely downloadable at www.netlib.org.

value for the aggregate shock z and a value for the aggregate capital from the previous time period, I can compute all new decisions on investment, consumption and employment by simulating income shocks. This new simulation gives a new value for the aggregate capital, which coupled with a new value drawn for the aggregate shock implies that I move forward to the next time period.

It is important to have a large number of agents for each age group. I used 250, which gives a total number of agents in the economy equal to 15,000. However, to minimize the error I "enforced" the law of large numbers by making sure that the fractions of labor income levels implied by (19) matched the theoretical ones, by randomly adjusting the values of the shocks.

5. Once an aggregate time series has been computed, $a(z)$ and $b(z)$ in (15)-(16) are estimated by least squares.

6. If these new values for $a(z), b(z)$ are "close" to the initial ones, an equilibrium has been found. If not update the new values by setting (for an arbitrary iteration j),

$$a(z)_{NEW} = \phi a(z)_{OLD} + (1 - \phi)a(z)_j$$

$$b(z)_{NEW} = \phi b(z)_{OLD} + (1 - \phi)b(z)_j$$

for $\phi \in (0, 1)$, and return to Step 2.

The forecasting rule for capital was of the form:

$$ln(K)_{t+1} = a_k(z_t) + b_k(z_t)ln(K)_t + \epsilon_{t+1} \;\; \epsilon_{t+1} \sim N(0, \sigma_k^2(z_{t+1}))$$

and for labor:

$$ln(N)_t = a_n(z_t) + b_n(z_t)ln(K)_t + \epsilon_{t+1} \;\; \epsilon_{t+1} \sim N(0, \sigma_n^2(z_{t+1}))$$

6.2.1 Accuracy Evaluation

I assessed the accuracy of the computed individual policy functions by measuring the error made in the Euler equations when they are evaluated at points different from the ones used to compute the solution. Specifically, I computed the Euler equation errors over points in a grid of capital and house holdings and aggregate capital. The grid essentially covered the same range for each variable as the grid used to compute the policy functions. It did not include the extreme values (smallest and largest) and it contained almost three times as many points (14,400). The following table shows average and maximum errors for the no-borrowing and borrowing economies. Both models gave very similar results: [17]

Table 8: Euler Equation Errors

Constraint	Average Error (%)	Maximum Error (%)
$1-\gamma=0$	0.03	154
$1-\gamma=0.9$	0.03	156

Krusell-Smith's approach relies on agents making small errors when forecasting prices using only aggregate capital to summarize the wealth distribution. For this reason, it is important to measure the accuracy of the aggregate laws of motion for capital and for employment. Table 9 shows the average and maximum absolute errors (in percent) between the forecast and actual capital (or employment).

Table 9: Aggregate Law of Motion Errors

	Average Error (%)	Maximum Error(%)
K' Regression, Tight Constraint	0.44	2.84
N Regression, Loose Constraint	0.04	0.18
K' Regression, Tight Constraint	0.53	3.24
N Regression, Loose Constraint	0.04	0.19

I also performed a test in the spirit of the one proposed by Den Haan and Marcet (1994). I obtained residuals from the two K' regressions and tested the hypothesis that the solution is exact by computing the test statistic (see their paper for the details of the procedure)

[17] Although maximum errors are somewhat large, the combinations of capital and house holdings that imply these errors may not have occurred in the simulation of the model economy. For instance, they could happen for a very small holding of capital the year prior to retirement, a very unlikely value since at that age all agents hold large amounts of capital.

using a constant and the amount of capital held by the 6 youngest generations as instruments. Under the null hypothesis, the test statistic is distributed as a χ^2_2. The values of the test statistic were 1.429 for the no-borrowing economy and 19.680 for the borrowing economy. The 0.05 and 0.95 cutoff points in a χ^2_2 are 0.103 and 5.99 respectively. Although it is the result of one simulation only, accuracy in the no-borrowing economy is high enough to be in the no-rejection region for the specified confidence level. For the no-borrowing economy, it is not. The number of observations used to perform this test was 2300.

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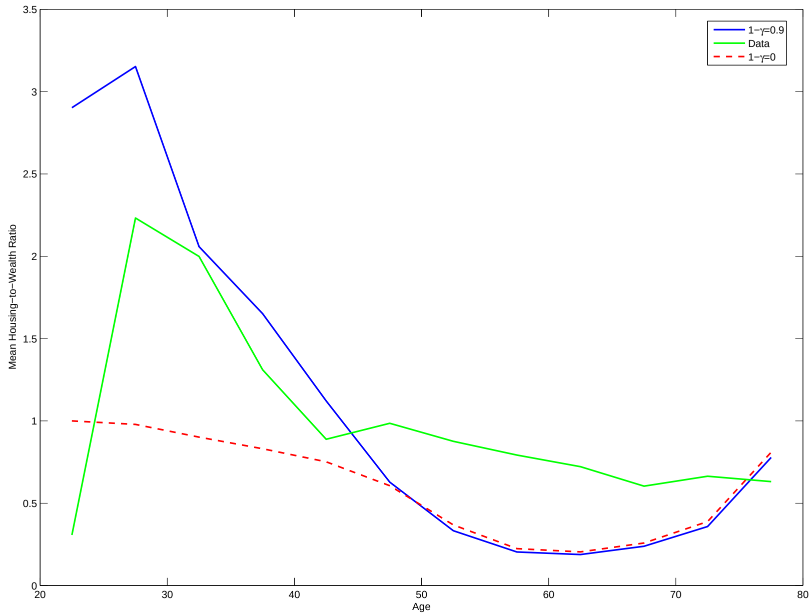

Figure 2: Model Economies versus Data

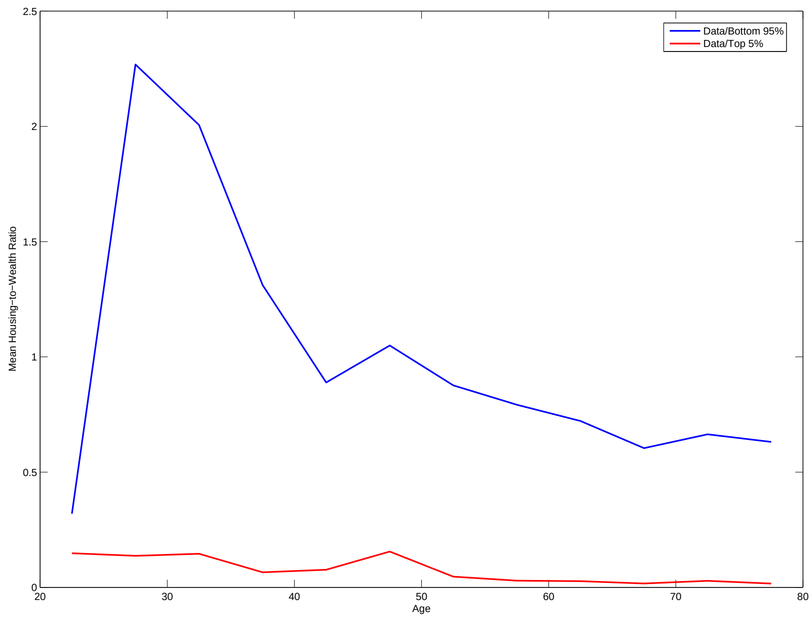

Figure 3: Housing to Wealth Ratio by Level of Wealth, Source: Survey of Consumer Finances

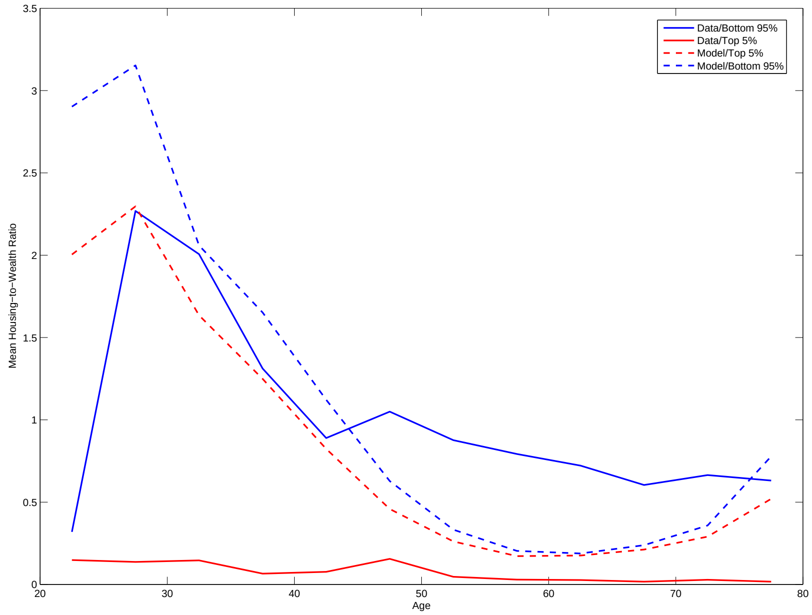

Figure 4: Housing to Wealth Ratio by Level of Wealth, $1 - \gamma = 0.9$

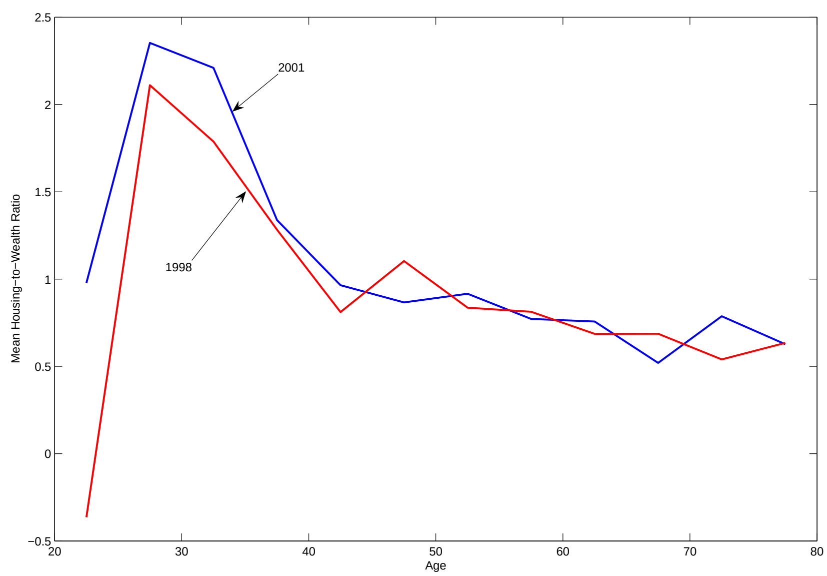

Figure 5: Housing-to-Wealth Ratios, Source: Survey of Consumer Finances (1998 and 2001).

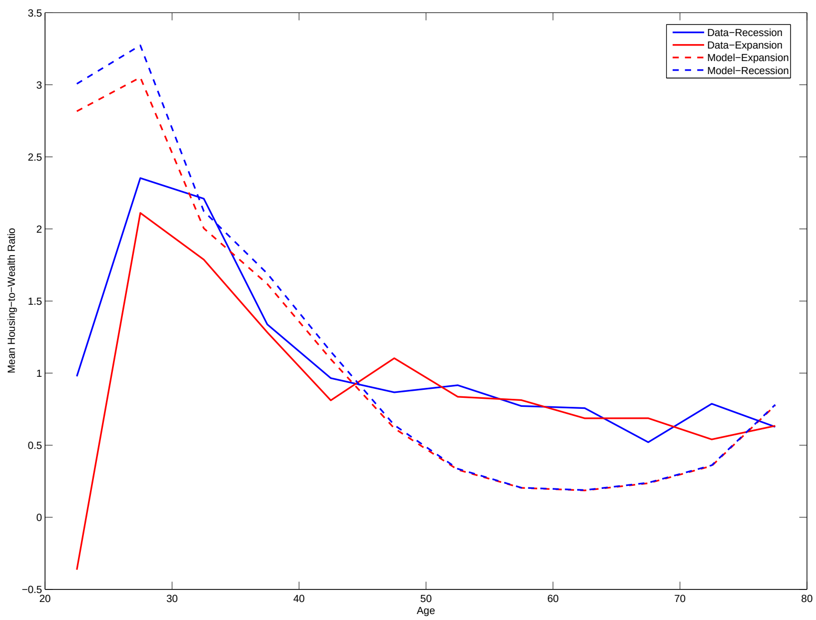

Figure 6: Housing-to-Wealth Ratios, $1 - \gamma = 0.9$

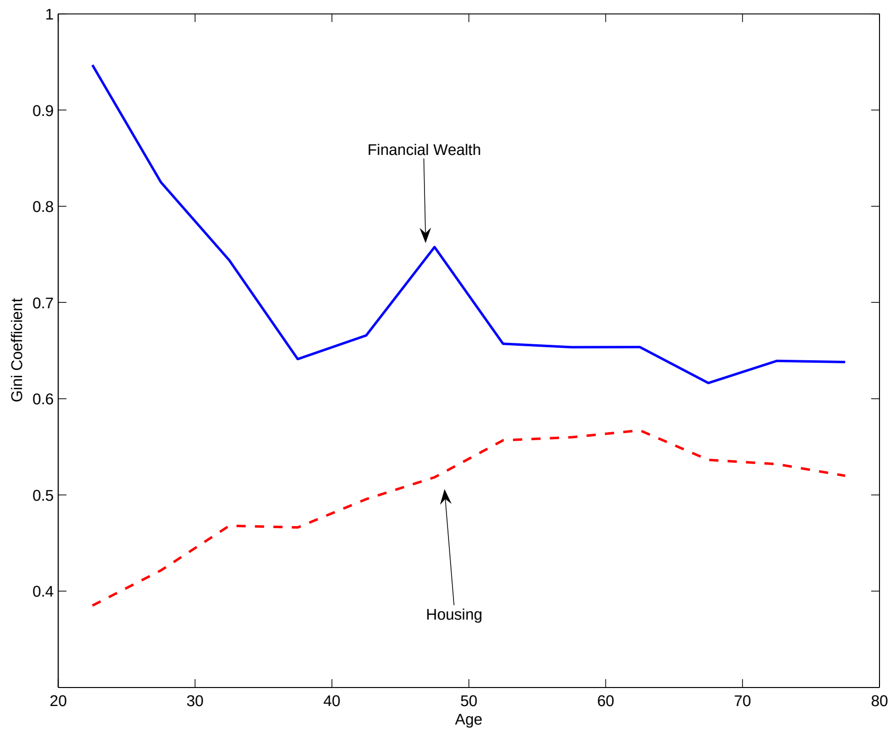

Figure 7: Gini Coefficients over the Life Cycle by Type of Wealth