Housing Tenure and Wealth Distribution in Life-Cycle Economies

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Abstract

Common practice in the housing and wealth distribution literature has proceeded as if the modeling of housing rental markets was unnecessary due to renters' relative low levels of wealth and the smaller fraction they represent in the total population. This paper shows, however, that their inclusion matters substantially when dealing with wealth concentration over the life-cycle. Renters are concentrated in the poorer and younger groups and in terms of the wealth inequality over the life cycle the model improves relative to a one asset economy and relative to a housing model with no rental markets.

Keywords: Wealth concentration, life cycle, housing tenure

JEL Codes: E21, D30

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1 Introduction

The comparison between the characteristics of the wealth distribution that results from equilibrium models and its properties in US data has been the subject of an extensive literature in macroeconomics. Aside from a few exceptions, housing wealth has rarely been modeled explicitly but has been lumped together with other assets in the "capital" stock, following the tradition of one-sector stochastic growth models. Even when housing has been separated from other assets (such as in Díaz and Luengo-Prado (2003) or Gruber and Martin (2003)) the existence of a real estate rental market has been completely ignored. While it is true that renters are a minority in the total population, they represent an overwhelming majority among the poorer and younger age groups. As I show below, their inclusion in dynamic macroeconomic models has important implications for the wealth concentration over the life cycle.

The modeling strategy is close to Huggett (1996), introducing some of the elements that he considers necessary for models to succeed in matching wealth distribution moments, but separating housing from the remaining capital stock and allowing agents to consume housing services through renting. It is an overlapping generations economy in which agents are subject to idiosyncratic income risk. Income is taxed and the proceeds finance a pay-as-you-go Social Security system which provides pensions for retired workers.

I focus on how the introduction of a rental market for housing affects the degree of wealth inequality over an agent's lifetime. I compare the model with housing tenure choice to two different economies. The first is the standard model, in which agents hold one asset and rent housing services in a rental market. The second is the (standard) housing model, in which everybody is a homeowner and rental markets for housing simply do not exist. The existence of some renters in all age groups has an impact on wealth inequality even when focusing only on total wealth: it increases wealth concentration for all ages improving over the standard model and the model with only homeowners. Regarding housing wealth, the (standard) housing model implies a flat profile for wealth concentration over an agent's lifetime. In the data, the profile is decreasing, just as it is for non-housing wealth. The model with housing tenure choice is also consistent with this fact.

In addition to the already mentioned work by Díaz and Luengo-Prado (2003) and Gruber and Martin (2003) in which they introduce durable goods into Aiyagari's (1994) model, there are a few other related studies that are worth mentioning for their close relation to the research presented here. The first is an article by Gervais (2002) on which the modeling of the housing rental market in this paper builds. He presents a deterministic economy in which agents have the choice of renting housing services or owning real estate capital, and analyzes the welfare implications of different taxation schemes. Platania and Schlagenhauf (2000) construct a life-cycle model where agents are subject to idiosyncratic risk and study the asset allocation problem between business capital and housing. These agents also have the choice between renting housing services in a rental market and owning housing capital. In their model, individuals are constrained to hold a fixed amount of housing and it is therefore inappropriate to study wealth distribution, or even portfolio choice, issues. Finally, there have been two recent studies that in modeling strategy and topics explored are closely related as well. Díaz and Luengo-Prado (2006) build a stochastic overlapping generations economy to study the effect that distortions in the housing market have on the wedge between the user cost of housing and the price paid in a rental market. Lastly, Chambers, Garriga and Schlagenhauf (2005), analyze, in a framework similar to the one I use here, the interactions between the tenure decision within an asset allocation problem for the household.

2 The Model

The economy consists of a continuum of agents with a total measure of one that live for I periods. Agents are born with zero wealth, and work during the first T periods of their lives. Retirement is mandatory at the end of period T and people live off their accumulated wealth for the remaining $TR = I - T$ periods.

Individuals maximize their expected lifetime utility over non-housing consumption (c) and housing services (s):

$$U(c, s) = E \sum_{i=1}^{I} \beta^{i-1} \phi_i u(c_i, s_i) \tag{1}$$

In the previous expression, the time-discount factor is denoted by β and the conditional probability of surviving from age i to $i + 1$ is denoted by ϕ_i.

2.1 Social Security

The government runs a "pay-as-you-go" system that taxes the labor (and capital income) of the younger generations (workers) and transfers resources to the older generations. A fixed level of Social Security benefits, denoted by b, is provided for the remaining lifetime of an agent.

2.2 Technology

There is an aggregate technology operated by a representative firm that produces output in this economy using capital K and labor N:

$$Y = F(K, N) \tag{2}$$

This production function satisfies the usual properties, increasing in both arguments, strictly concave and homogeneous of degree one. Output can be costlessly allocated to consumption, business capital investment and investment in residential capital.

The technology for transforming residential capital (h) into housing services (s) is linear: $s = h$, where s is the amount of services enjoyed by an individual having h units of real estate capital. These services can be obtained by either owning residential capital or by renting them in a housing rental market. Both renting and owning are mutually exclusive. In an attempt to mimic fiscal policy in several developed nations, homeownership is "subsidized" in two ways: first, the ownership of real estate is not taxed (i.e. implicit rents are not taxed), and second, mortgage interest rate payments are deductible. This deductability has the form of "tax credit": mortgage holders pay an effectively lower interest rate on their loans than the market interest rate. Hence, in the absence of any additional friction agents would rather own than rent. However, housing capital is not perfectly divisible: if an agent wants to own she needs to buy a house of at least size $\underline{h}$. If she can not afford it, she must enjoy housing services by renting. The financing of a home purchase is done by entering in to a one period mortgage. Agents are only allowed to borrow at most a fraction $1 - \gamma$ of the value of the new home. Hence, one can think of γ representing the downpayment fraction. There is no difference in depreciation between rented capital and owned capital, with all housing capital depreciating at a rate of δ_h per period.

Agents supply inelastically whatever amount of time they are endowed with. However, they are subject to productivity shocks that alter their level of efficiency. The structure of financial markets is such that agents can not trade directly contingent claims to hedge against shocks to labor productivity and the smoothing of income fluctuations is done by adjusting the holdings of capital and residential stocks.

2.3 The Agent's Problem

Aside from the usual choices of allocating consumption and savings, and allocating total investment between residential and business capital, agents have to decide whether they want to rent housing services or they want to own residential capital stock. As usual, to finance both types of consumption, individuals obtain income by inelastically supplying one unit of labor and from renting business capital.

Denote by $V_j(a, \xi)$ the value function of an agent belonging to generation j that enters the current period. The state variables for this optimization problem are the level of cash-on-hand (a), which comprises the level of capital (k), its interest (rk) and the undepreciated real estate holdings $(1-\delta_h)h$; and the value of the productivity shock in the previous period (ξ). The consumer will compare the values of renting home services the current period versus purchasing a home. Denote these two values by V^R and V^O respectively. Then, for an age j individual:

$$V_j = max\{V_j^R, V_j^O\} \tag{3}$$

I now define separately the two problems that determine V_j^R and V_j^O. The value of renting, V_j^R is determined by solving the problem:

$$V_j^R = \max_{k', s, c} \{u(c, s) + \beta E V_{j+1}(k', \xi')\} \tag{4}$$

$s.t.$

$$c + ps + k' \leq a + y \tag{5}$$

$$k' \geq 0 \tag{6}$$

and

$$a' = (1 + r(1-\tau))k' + (1-\delta_h)h \tag{7}$$

Equation (4) states that agents choose housing services, consumption and asset holdings to maximize lifetime utility. Equation (5) is the budget constraint: the sum of expenditures on consumption, rented housing services and investment cannot exceed the sum of cash-on-hand and other income y. This income equals b, the pension benefits, if the agent is retired and $(1-\tau)we_j\xi$ if the agent is of working age. Notation is standard. Prime variables denote next period values, w is the wage, and e_j is an age-j-specific efficiency factor. Expectations are taken with respect to the distribution of the productivity values ξ.

Analogously, the value of becoming a homeowner is the solution to:

$$V_j^O = \max_{k', h', c} \{u(c, s) + \beta E V_{j+1}(a', \xi')\} \tag{8}$$

s.t.

$$c + h' + k' \leq a + y \tag{9}$$

$$s = h' \tag{10}$$

$$k' \geq -(1-\gamma)h' \tag{11}$$

$$a' = (1 + r(1-\tau))k' \tag{12}$$

$$h' \geq \underline{h} \tag{13}$$

In the homeowners' problem, the borrowing constraint (11) is a downpayment constraint: the agent can not borrow more than a fraction equal to $(1-\gamma)$ of the house she wants to buy. In addition, the indivisibility constraint (13) states that any house the individual buys must have a minimum size of $\underline{h}$.

2.4 Timing

Households arrive with wealth k and in case they had chosen to be homeowners the previous period, a stock of housing equal to $(1-\delta_h)h$. After observing their productivity shock ξ they decide their tenure position knowing the amount of resources they have is the sum of the capital plus its interest, undepreciated real estate and wage rate, scaled by the productivity value. In case they decide to be renters, they provide labor services to the firm, rent their business capital and choose the amount x of housing services to enjoy and the capital holdings they will start next period with: k'. If they choose to purchase housing capital h', they do so and begin enjoying housing services from it immediately As in the case of renters, prior to that, they supply the representative firm with labor services and an amount of business capital to use in production.

2.5 The Rental Market

For expositional purposes, it is convenient to think about the rental market as being run by a rental agency/financial institution. The role of this financial institution is just to pool together assets from households. These assets are used to purchase new residential capital to be rented out in a housing rental market or to issue loans to potential homeowners. Loans have an interest rate equal to r and rented housing services are charged at a price p. Assets held at the financial institution are remunerated at the rate r.

The equilibrium rental rate of housing capital p is such that leaves households (and hence the financial institution) indifferent between renting housing services or becoming a homeowner, in the absence of fiscal policy frictions and indivisibility constraints. This rate is given by $p = \frac{r+\delta_h}{1+r}$[1]. Denote by H^b the amount of residential capital owned by the financial institution, and which is therefore rented out, and by D^b the amount of deposits that the financial institution obtains from households.

[1]This price differs from the usual $p = r + \delta_h$ by a "discount factor" $\frac{1}{1+r}$. This follows from the unconventional timing assumed in this paper: the residential capital can be rented away immediately after purchase, and it is not subject to the one period time-to-build constraint present in business capital. The derivation of this rental price is through a straightforward manipulation of the Euler equations.

2.6 Equilibrium

Denote by z the vector of state variables (a, ξ) and by μ_j the measure of agents of age j. A stationary equilibrium is a set of decision rules for consumption $c(z, j)$, financial asset holdings $k(z, j)$, real estate holdings $h(z, j)$ and rented housing services $x(z, j)$, prices w,r, age-dependent distributions across wealth and income levels $\Psi_1, \ldots, \Psi_I$, a tax rate τ, level of benefits b, and aggregate quantities K, N, D^b, H^b such that:

1. Decision rules are optimal.

2. Prices are determined competitively:

$$r = F_1(K, N) - \delta_k \tag{14}$$

$$w = F_2(K, N) \tag{15}$$

3. Asset markets clear:

$$K = \sum_{j=1}^{T+TR} \mu_j \int_Z k(z, j)) d\Psi_j - H^b \tag{16}$$

$$D^b = \sum_{j=1}^{T+TR} \mu_j \int_Z k(z, j) \chi_{\{k:k>0\}}(k) d\Psi_j - K \tag{17}$$

4. Rental market clears:

$$H^b = \sum_{j=1}^{T+TR} \mu_j \int_Z x(z, j) d\Psi_j \tag{18}$$

5. Goods market clears:

$$F(K, N) = \sum_{j=1}^{T+TR} \mu_j \int_Z (c(z, j) + \delta_h h(z, j)) d\Psi_j + \delta_k K + \delta_h H^b \tag{19}$$

6. The government balances its budget in every period.

$$b \sum_{j=T+1}^{T+TR} \mu_j = \tau w N + \tau \sum_{j=1}^{T+TR} \mu_j \int_Z k(z, j)) d\Psi_j$$

The equilibrium definition is standard and most equations are straightforward [2]. However, two equations deserve further explanation: the capital market clearing condition (16) and the rental market clearing condition (18). To compute aggregate capital K the stock of rented residential capital needs to be subtracted from the total amount of financial assets held by individuals. Notice that k can be negative therefore it includes the amount agents borrow from banks. The rental market clearing condition just states that the amount of residential stock owned by financial institutions H^b needs to be equal to the amount of services rented by individuals.

2.7 The Housing Tenure Decision

The following result summarizes the housing tenure decision.

Result 1 *If savings are large enough so that neither the downpayment or the minimum size constraints bind, owning is preferred to renting.*

The intuition behind this result is the as follows. An individual with a small level of savings who cannot afford the downpayment for a house needs to enjoy housing services through renting. Assume now that the level of savings is large enough so that neither the downpayment (11) or the minimum size (13) constraints bind. An individual has a choice between purchasing a house of a particular size h or renting it and investing in business capital. At the time of the home purchase an individual forgoes $1-p$ units of consumption today and gets $1-\delta_h$ units of consumption the following period. Therefore $\frac{1-\delta_h}{1-p}$ is the "return" (per unit of consumption good) of investing in a home. If she rents, the return in the capital market (per unit of consumption good) is $1+r(1-\tau)$. Since

[2] Note that the measures of the different age groups are independent of the wealth distribution and therefore can be outside the integral sign.

$\frac{1-\delta_h}{1+r} > 1 + r(1-\tau)$ the individual is better off by purchasing a home. One can clearly see that what drives the result is the differential tax treatment of the two types of capital. If the government were to tax the flow of real estate services (the implicit rent) at the same rate as it taxes business capital income, the result would not hold.

3 Parameterization

3.1 Demographics

The model period is set to be equivalent to 5 years. Agents are assumed to be born being 21 years old. Individuals live for $I = 12$ periods (60 years), and retire after 9 periods.

3.2 Social Security

The government taxes (at a common rate τ) labor and capital income. A fraction of these resources is used to finance a social security system that provides life long fixed level of benefits b to retired individuals. The tax rate on income is set so that, given all the other parameter values, the level of benefits is approximately 30% of the average wage.

3.3 Preferences, Technology and Endowments

The utility function chosen is of the constant relative risk aversion class, standard in the wealth distribution literature, with a Cobb-Douglas aggregator between housing services and non-housing consumption:

$$u(c, s) = \frac{(c^\theta s^{1-\theta})^{1-\sigma}}{1-\sigma} \tag{20}$$

Parameters in the model were chosen to match some features of the United States economy during the last forty years. The discount factor β was chosen to match a business capital to output ratio of 1.8. The coefficient of relative risk aversion σ was set to 2.5, roughly in the middle of the range of values used in the literature.

The parameter θ - the share of non-housing consumption in the utility function - was set at 0.8. This value is consistent with housing expenditures being about 20% in the Consumer Expenditures Survey (see Peterson (2003)). Regarding the unit elasticity of the between housing services and non-housing consumption, it is not an unreasonable assumption based on empirical evidence (see Fernández-Villaverde and Krueger (2002)). Nevertheless, below I provide some sensitivity analyses to changing this assumption.

3.3.1 The Earnings Process

When looking at the properties of the wealth distribution, it is important to have an accurate approximation to the labor earnings process. Huggett (1996) estimates an AR process for the logarithm of the labor endowment:

$$z_t = \rho z_{t-1} + \epsilon_t \tag{21}$$

The disturbance term ϵ is distributed normally with mean zero and variance σ_ϵ^2. Huggett sets $\sigma_\epsilon^2 = 0.045$, with a persistence parameter chosen so that the unconditional variance is equal to 0.38, which in turn implies a Gini coefficient in wage earnings of 0.42. This resulted in a value for ρ of 0.96.

For computational purposes I have approximated this process as a seven state Markov chain. In the Appendix I present a comparison of the moments implied by the continuous and the discrete state processes. The transformation of the continuous state process into its five-year equivalent was done prior to its conversion into a discrete state process.

In addition to this idiosyncratic productivity shock agents face an age-dependent efficiency profile $\{\eta_i\}_{i=1}^{I}$ used in Huggett and Ventura (1999) [3]. Hansen (1993) estimated a

[3] I thank Mark Huggett for making the data available.

labor endowment efficiency profile from the Current Population Survey (CPS) for different age groups. This profile is used to scale median wages over an agent's working life.

3.3.2 Technology

Output is produced by combining capital and labor through a Cobb-Douglas production function:

$$Y = K^{\alpha} N^{1-\alpha} \tag{22}$$

The value chosen for α was 0.30 which implies a share of labor in total income of 0.70, roughly consistent with US data and very close to the value used in Greenwood et al. (1995), 0.29. The depreciation rates were obtained from the data using conditions relating investment, capital and output in the steady state. For both types of capital, business and residential, in a steady-state:

$$\frac{I_k/GDP}{K/GDP} = \delta_k \tag{23}$$

$$\frac{I_h/GDP}{H/GDP} = \delta_h \tag{24}$$

US data from 1964-2003 implies values for δ_h and δ_k of 4.3% and 9.4% per year. The equivalent five-year values were $\delta_h = 1-(1-0.043)^5 = 0.197$ and $\delta_k = 1-(1-0.094)^5 = 0.3895$.

The structure of the housing market implies that in order to be a home owner, the size of the purchase must be equal or larger than $\underline{h}$. The value for this parameter was chosen so that the model would imply a ratio between the owned housing stock and the rented housing stock equal to approximately 3. The resulting value was approximately 40% of output.

In the US economy a typical value for downpayment fractions of the house value at

Table 1: Summary of Parameter Values

Parameter / Variable	Value	Target / Source
β	0.931	$\frac{K}{Y} = 1.8$
$\underline{h}$	0.410	$\frac{H_O}{H_R} = 3$
τ	0.035	$b/w = 0.3$
θ	0.800	20% exp. in housing; Peterson (2003)
z	—	Huggett and Ventura (1999)
e	—	Huggett and Ventura (1999)
α	0.30	NIPA
δ_k	0.094	"
δ_h	0.043	"
$1-\gamma$	0.8	Fernández-Villaverde and Krueger (2002)

the time of the purchase is 20% (see Fernández-Villaverde and Krueger (2002)). For this reason the borrowing constraint is specified so that agents can borrow up to 80% of the house they want to buy: $1 - \gamma = 0.8$.

Table 1 summarizes the parameterization for the model, stating values for parameters and their target/source. Details about the solution of the model are included in the Appendix, but the methods are fairly standard. Once decision rules are obtained, summary statistics are computed by simulating life-cycle paths for a large number of agents drawing shocks from the appropriate (discretized) distribution for productivity shocks.

4 Results

Table 2 shows some aggregate annual statistics[4] for the United States economy during the period 1964-2003[5].

The table shows the importance of housing in the aggregate economy, with residential stocks representing about half of the entire capital stock. The total capital to output

[4]The appendix provides definitions of all variables used throughout the paper.
[5]The last entry of the table corresponds to data from 1987 to 2003.

Table 2: US data (1964-2003)

Variable	Average
$(K+H)/GDP$	3.471
K/GDP	1.754
H/GDP	1.717
$K/(K+H)$	0.505
$H/(K+H)$	0.495
H_O/H_R	2.96

Table 3: Model Output, Averages

Variable	Model (Year)
$(K+H)/GDP$	3.549
K/GDP	1.754
H/GDP	1.795
$K/(K+H)$	0.494
$H/(K+H)$	0.506
H_O/H_R	2.957

ratio, $\frac{K+H}{GDP}$, seems somewhat larger than values previously reported, for example in Cooley and Prescott (1995), but the definition of GDP used in this paper does not include housing services, which account for about 10% of output. The last cell in Table 2 gives the ratio of the stock of residential capital that is owned to that which is rented. In the US economy the average for this ratio is 2.96.

Table 3 gives the model's values for the aggregate statistics included in Table 2. To facilitate the comparison I present the results in per year equivalents. Two important statistics were not targeted by the model: the proportion of housing wealth in the economy's total wealth and the ratio of the housing stock to output. In both those dimensions the model matches the averages observed in the data quite closely.

Table 4: US Economy: Gini Coefficients

Variable	US Pop.	21-25	26-30	31-35	36-40	41-45	46-50
Total Wealth	0.70	1.00	0.84	0.79	0.71	0.66	0.720
Primary Residence	0.66	0.88	0.78	0.74	0.66	0.63	0.62
Variable	51-55	56-60	61-65	66-70	71-75	76-80	—
Total Wealth	0.66	0.65	0.65	0.60	0.58	0.60	—
Primary Residence	0.63	0.62	0.62	0.60	0.59	0.61	—

Data on households' wealth come from the Survey of Consumer Finances (SCF) . The SCF has become the main source used by financial economists to address any question related to the composition of balance sheets in US households. I have used the 2001 version in which a total of 4,400 families were interviewed. From the SCF, for a given age group, one can compute measures of concentration for any asset. I focus here on total wealth i.e. networth and the value of the primary residence.

Table 4 presents data on Gini coefficients for total wealth (including housing) and the value of the primary residence for the overall population and the different age groups. The Gini indices for total wealth over the life cycle follow a decreasing pattern (a decreasing level of concentration), due to a large number of net borrowers in the younger age groups. It starts with a value of unity for the first age group (21-25), decreasing to about a value of 0.60 for the older generations. For a similar reason, the larger concentration of renters in the younger age groups, the Gini index for the value of the primary residence is also decreasing. Levels of concentration, however, are smaller than for total wealth. The largest coefficient occurs also for the younger age group with a value of 0.88, decreasing to a value of about 0.60 for the older age groups. For the entire population the Gini for total wealth is 0.04 larger than for the value of the house (0.70 vs. 0.66).

Before I analyze the implications for housing wealth, the implications of the model for total wealth gives rise to interesting comparisons across different versions of the artificial economy. In particular, it is interesting to compare the three "basic" versions: the "Standard" model (i.e. a model in which the only wealth households hold is business capital k and everyone obtains housing services through the rental market), the "Tenure" model, which is the baseline version and it is parameterized as described above, and finally the "Homeowners" models in which everyone owns the housing capital they enjoy housing services from. The "Homeowners" model can be obtained by brute force (by shutting off the rental market) or endogenously by setting $\underline{h}$ to zero. Analogously, the "Standard" model can be obtained by not allowing homeownership or by setting $\underline{h}$ to a very high value. Figure 1 shows the levels of concentration, as measured by the Gini coefficient, for total wealth over an individual's life cycle for four different economies. The large-dotted line represents the US economy in 2001. The other three lines are artificial economies. The solid line is the "Standard" model, the dash-dotted line is the "Tenure" model, and finally the small-dotted line is the "Homeowners" model.

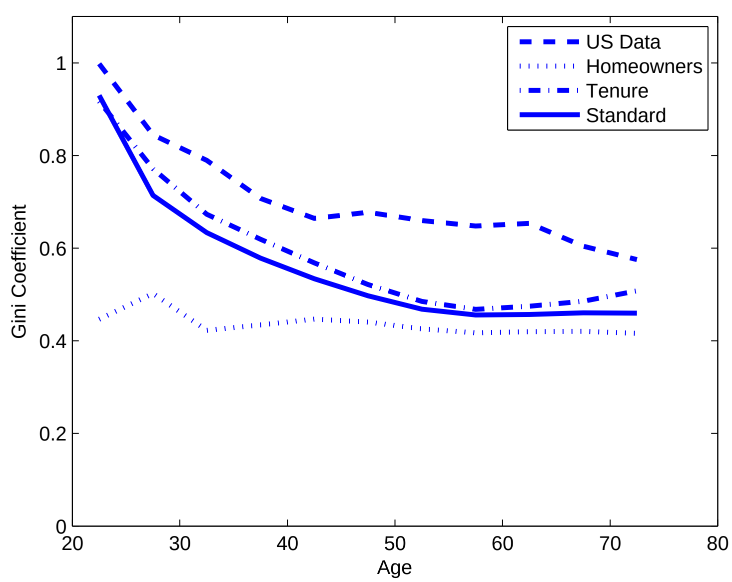

Figure 1: Gini Coefficients: Total Wealth.

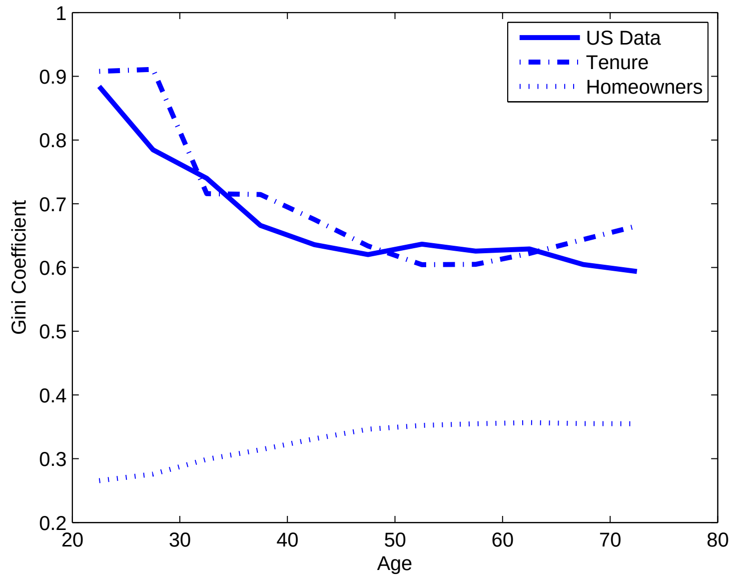

Figure 2: Gini Coefficients: Housing.

As already discussed above, the US economy displays a decreasing level of wealth concentration as individuals age [6]. The slope of the decrease in concentration is high for the younger age groups and it decreases as the agents age. Only two artificial economies are consistent with this pattern. The "Standard" and the "Tenure" models. The concentration of total wealth in the "Homeowners" model is flat over the life cycle and the Gini coefficients are smaller for all age groups than in the other two economies. Moreover, levels of concentration for the "Tenure" model are closer to their empirical counterparts than in the "Standard" model. Why the flat profile in the "Homeowners" model? The reason is simple: consumption of housing services is all done via homeownership, increasing the wealth of the poorer agents (everybody needs to consume housing services, after all) and decreasing the variance of the wealth distribution for any given age group.

Turning now to housing wealth, it is evident from Figure 2 that the "Homeowners"

[6]Notice that the usual caveat applies here: I am focusing on a cross-section of individuals for a given year and not tracking individuals over their lives

model is a lousy approximation to US data. It not only understates the degree of wealth concentration across all age groups, but it misses the overall pattern as well: Gini coefficients stay relatively flat for all generations, and in fact they are lower for the younger age groups. The explanation for this last result is that housing wealth is restricted to be positive. The dispersion in the younger age groups is the result of different house sizes agents can purchase with their different income endowments. The persistence of the income process makes agents more unequal as they age, and therefore the dispersion of housing wealth increases as well. When renters are present, there exists a significant mass of agents with zero housing wealth in the younger age groups. As agents age, the larger realized income dispersion should increase the level of housing wealth concentration. However, the higher financial wealth that most agents accumulate as they get older permits individuals to become homeowners, decreasing that point mass at zero housing wealth. This second effect dominates the first, resulting in decreasing housing wealth concentration levels over the life cycle when rental markets exist.

A potentially important assumption in the above parameterization is the unit elasticity between housing services and non-housing consumption. Unfortunately, there is little agreement among economists on the value of this elasticity. Fernández-Villaverde and Krueger (2002) report that attempts to estimate it give widely different results depending on sample composition or how preferences are specified. In many cases, standard errors are large and as a consequence it is difficult to reject the hypothesis of a value of one. Nevertheless, it can be illustrative to check the sensitivity of the results to the Cobb-Douglas assumption by specifying a CES aggregator between the two types of goods. With this change the per-period utility function now reads:

$$u(c, s) = \frac{\{\theta c^{\rho} + (1-\theta)s^{\rho}\}^{\frac{1-\sigma}{\rho}}}{1-\sigma}$$

Without changing the values of the remaining parameters, I have obtained results for four values of ρ: -0.4,-0.2,0.,0.2. Values of ρ smaller than zero imply less substitutability between the two goods relative to the baseline parameterization. Figures 3 and 4 show concentration levels over the life-cycle for total wealth and for the value of the primary residence, for those four values of ρ.

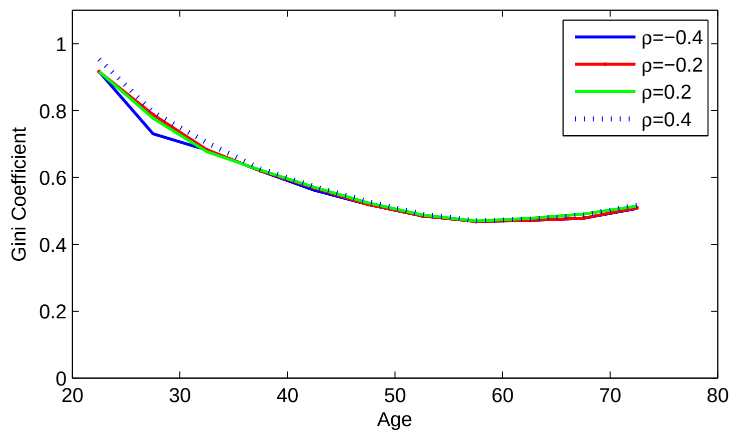

Figure 3: Gini Coefficients: Total Wealth.

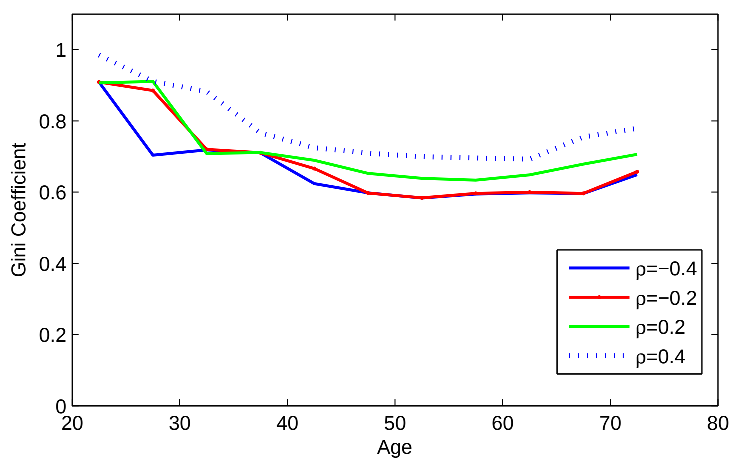

Figure 4: Gini Coefficients: Housing.

The implications for total wealth barely change. The four lines are almost indistinguishable and the degree of concentration is very close to the values obtained for Cobb-Douglas preferences. In the case of housing wealth, the differences are larger. In fact the large substitutability ($\rho = 0.4$) overstates the degree of concentration, on average, by 0.1 per age group. On the other extreme, the (relatively) large complementarity case only overstates it by 0.03 on average per age group. However, the overall message does not rely upon the assumption of a unit elasticity between the two goods.

5 Conclusion

Several examples in the macroeconomics literature have focused on models of housing to study portfolio choice (e.g. Silos (2006)), wealth distribution (Gruber and Martin (2003) and Díaz and Luengo-Prado (2003)) or consumption (Fernández-Villaverde and Krueger (2002)). Neither study has modelled rental markets for housing services and in their models every agent is a homeowner. I have shown here that the introduction of rental markets is both qualitatively and quantitatively important when the focus is on the degree wealth concentration over the life cycle. First, regarding total wealth the model compares favorably against both the standard model, by which I mean a model with one type of wealth, and a model in which everyone is a homeowner, as the ones mentioned above. Second, with respect to housing wealth it shows how to match not only the degree but also the shape of wealth concentration over the life-cycle the introduction of a rental market is needed.

References

[1] Aizcorbe, A., Kennickell, B. and Moore, K. (2003), "Recent Changes in U.S. Family Finances: Evidence from the 1998 and 2001 Survey of Consumer Finances", *Federal Reserve Bulletin*, vol. 89, pp. 1-32.

[2] Aiyagari, R. (1994),"Uninsured Idiosyncratic Risk and Aggregate Saving", *Quarterly Journal of Economics*, Vol. 109, No. 3, pp. 659-684.

[3] Chambers, M., Garriga, C. and Schlagenhauf, D., (2005),"Housing Tenure and Investment Decisions", manuscript, Florida State University.

[4] Cooley, T. and Prescott, E. (1995), "Economic Growth and Business Cycles", in *Frontiers of Business Cycle Research*, Cooley, T. (ed.), Princeton University Press.

[5] Díaz, A. and Luengo-Prado, M., (2003), "Precautionary Savings and the Wealth Distribution with Durable Goods", manuscript, Northeastern University.

[6] Díaz, A. and Luengo-Prado, M., (2006), "On the User Cost and Homeownership", manuscript, Northeastern University.

[7] Feigenbaum, J., (2005), "Can Mortality Explain the Consumption Hump?", manuscript, University of Pittsburgh.

[8] Fernández-Villaverde, J. and Krueger, D. (2002) "Consumption and Saving over the Life Cycle: How Important are Consumer Durables", Stanford Institute for Economic Policy Research Discussion Paper 01-34.

[9] Gervais, M., (2001), "Housing Taxation and Capital Accumulation", *Journal of Monetary Economics*, Vol. 49 (7), pp. 1461–1489

[10] Greenwood, J., Rogerson, R., and Wright, R. (1995),“Household Production in Real Business Cycle Theory”, in *Frontiers of Business Cycle Research*, Cooley, T. (ed.), Princeton University Press.

[11] Gruber, J. and Martin, R. (2003), “Precautionary Savings and the Wealth Distribution with Illiquid Durables”, International Finance Discussion Paper Number 773, Board of Governors of the Federal Reserve System.

[12] Hansen, G. (1993), “The Cyclical and Secular Behavior of the Labor Input: Comparing Efficiency Units and Hours Worked”, *Journal of Applied Econometrics*, Vol. 8, pp. 71-80.

[13] Huggett, M. (1996), “ Wealth Distribution in Life-Cycle Economies”, *Journal of Monetary Economics*, 38(3), pp. 469-494.

[14] Huggett, M. and Ventura, G. (1999), “On the Distributional Effects of Social Security Reform”, *Review of Economic Dynamics*, Vol. 2, pp. 498-531.

[15] Peterson, B., (2003), “Individual Uncertainty, Aggregate Uncertainty and the Housing Market”, manuscript, Indiana University, Department of Economics.

[16] Platania, J. and Schlagenhauf, D., (2000), “Housing and Asset Holding in a Dynamic General Equilibrium Model”, manuscript, Florida State University.

[17] Silos, P. (2006), “Housing, Portfolio Choice and the Macroeconomy”, *Journal of Economic Dynamics and Control*, forthcoming.

6 Appendix

6.1 Computational Details

The model is solved numerically and the solution procedure is standard. In a nutshell, after specifying grids for the state variables (cash-on-hand and productivity shocks), one solves for the optimal decision rules and value functions starting from age $T + TR$ to age 1. For every grid point on the state variables grids, for every age group and productivity shock level, one can solve for the optimal choice for business capital, housing services and tenure status. The level of housing services, when agents choose to be renters, is not discretized. However, I restrict the housing choice when homeownership is preferred to fall on a grid. However, the number of gridpoints is large enough so that the approximation is accurate. The procedure is summarized in the following steps:

Step 1: Guess a level of aggregate capital K.

Step 2: Given interest rates and wages implied by that level of capital, solve for the optimal decision rules regarding tenure status, capital holdings and housing services. This is done for all age and income groups.

Step 3: Simulate life cycle paths for consumption, investment, etc..., for a large number of agents. Compute aggregate capital.

Step 4: If the level of capital is close to the one guessed in *Step 1*, an equilibrium has been found. Otherwise, update capital and return to *Step 2*.

6.1.1 Approximation of the Earnings Process

The accurate approximation as a discrete state process of the continuous state autoregression for labor earnings is important because the characteristics of the earnings process greatly affect the model's output regarding the wealth distribution.

The approximation involved two steps. The first step involves transforming a yearly

Table 5: Earnings Approximation: Accuracy

Variable	Mean	Variance	Skewness	1st Autocorr.
z_t	0.003	0.584	0.054	0.815
z_t, discrete	-0.009	0.523	0.003	0.773
e^{z_t}	1.349	1.562	3.871	0.787
e^{z_t}, discrete	1.284	1.052	2.220	0.730

model into the 5-year frequency. This was done by simulating the yearly model and sampling every fifth element to construct the five year equivalent. The second step involves the discrete state approximation to this 5-year model. The number of states used in the approximation is 7. Computational constraints precluded the number to be larger although it would clearly be desirable.

The Table 5 provides a comparison of moments for z_t and e^{z_t}, (the income shocks), and their discrete approximations:

6.2 Data

6.2.1 National Accounting Data

Almost all of the aggregate data comes from the Bureau of Economic Analysis website (*www.bea.gov*). The only exceptions are the United States population, the average weekly hours worked and the number of employees in the private sector, all of which come from the Bureau of Labor Statistics Website (*www.bls.gov*). The data are annual (except when extracting the Solow residual, see below) starting in 1964 and ending in 2003.

- *Gross Domestic Product*: Output is defined as Gross Domestic Product minus Consumption Expenditures in Durable Goods minus Expenditures in Housing Services minus Net Exports minus Government Consumption and Investment Expenditures. Output was transformed into *per capita* terms through dividing by the US population and transformed into real terms by deflating using the GDP deflator.

- *Investment*: Aggregate investment is Total Gross Private Domestic Investment. Business investment is the sum of non-residential investment in structures, equipment and software. Residential Investment is Total Investment minus Business Investment.

- *Consumption*: Consumption is defined as Personal Expenditures in Consumption minus Expenditures in Durable Goods minus Expenditures in Housing Services. Investment and Consumption were also deflated by the GDP deflator and transformed into *per capita* terms through dividing by the US population.

- *Capital Stocks*: The stocks of both residential and business capital come from the Fixed Assets Tables (Current Net-Cost). The definition of Residential Capital is Residential Structures. Business capital is defined as Total Private Fixed Assets minus Residential Structures. Data on residential stocks by tenure (renters vs. owners) also come from the Fixed Assets Tables.

6.2.2 Household Wealth Data

As mentioned in the text, data on household's wealth comes from the Survey of the Consumer Finances. Total wealth is defined as Networth, which equals Assets minus Debt. Housing wealth is the value of the primary residence. This value is a subjective assessment of the