Risk and the Misallocation of Human Capital

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Abstract

With risk-averse workers and uninsurable earnings shocks, competitive markets allocate too few workers to jobs with high earnings uncertainty. Using an equilibrium Roy model with incomplete markets, we show that risky occupations are inefficiently small and hence talent is misallocated. We obtain analytical expressions for the compensation for risk in the labor market, and for the aggregate level of human capital and output. We also study the welfare properties by solving for the constrained-efficient allocation. Misallocation is positively related to the correlation between a worker's abilities in different occupations. Quantitatively we find that market incompleteness can by itself generate permanent output and welfare losses of close to one percent of output. Around 35% of the loss is due to the presence of the pecuniary externality.

Key words: Misallocation, Human Capital, Occupations, Risk, Incomplete Markets, Frèchet, Roy Model.

JEL Classifications: E21 · D91 · J31.

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1 Introduction

Misallocation of human capital lowers productivity. Occupation or industry-specific human capital is an important feature of labor markets. For example, many technical, medical and legal occupations require knowledge in a narrowly defined field. It is rarely possible to work in such occupations without first obtaining occupation-specific skills and credentials through specialized training. At the same time, due to technological progress, international trade or urbanization, workers in certain occupations are subject to permanent earnings shocks that are hard to predict when making decisions about investing in skills training. The fear of high potential losses arises because there are no private insurance markets to hedge against these shocks. These shocks displace workers that are heavily invested in occupation- or industry-specific human capital.

In this paper we are the first to study how incomplete markets shape the aggregate allocation of talent and aggregate output. We propose a Roy model and compare the allocation of talent and welfare between a *laissez faire* competitive equilibrium and a constrained-efficient equilibrium. By constrained efficiency we mean a planner's allocation who maximizes the population's average welfare but who is still constrained by market incompleteness. We provide a quantitative analysis to study cases in which shocks to workers' human capital are caused by policy (e.g. a trade reform) or by technological progress. We find that the misallocation caused *only* by market incompleteness produces permanent losses of almost one percent of output. Our results shed new light on the cost of market incompleteness and they can inform policymakers when designing policies aimed at providing earnings or unemployment insurance for workers.

Our general equilibrium Roy model features a labor market where workers self-select into an occupation or industry based on their comparative and absolute advantages.[1] We assume that workers are risk-averse and human capital (for example acquired through specialized training) is specific to an occupation or industry. Workers' occupational choices determine both the level of output and the wage distribution in the economy.

We solve the problem of a planner that cannot complete the market; the constrained efficient or second-best allocation. We show that relative to the competitive equilibrium, welfare rises in this economy. This welfare increase implies that there are pecuniary externalities that workers do not internalize. We focus on calculating the output costs of human capital misallocation. We leave aside the distributional concerns and thus we do not study the welfare implications of different tax and transfer schemes that could improve the competitive allocation.

Our model features two occupations (without loss of generality) and the choice of a career is based on two factors: (i) a worker's talents in each occupation, and (ii) each occupation's earnings uncertainty, measured by the variance of permanent shocks to earnings. Workers' talents are modeled as draws from a Frèchet distribution. We allow them to be correlated, so that we can distinguish between comparative and absolute advantages. One extreme case is that of perfectly correlated draws in which a worker's ability is the same across occupations (purely absolute advantage). The other extreme would be the case of independent draws (comparative advantage). The model's tractability allows us to obtain closed-form solutions for various outcomes of interest such as the allocation of workers, output, and the wage and earnings premia.[2] In addition, the tractability illustrates the mechanics of the interplay between abilities and risk in affecting allocations and output in a transparent way.

One crucial element of the economy involves the tradeoff between risk and out-

[1] Throughout we use the terms industry and occupation interchangeably. Although our empirical analysis focuses on industries, a large share of workers in a given occupation (with the exception of managers) works on only one or two sectors. For example, more than 50% of Salespersons are concentrated in Wholesale Trade and Retail Trade.

[2] By a wage or an earnings premium we refer to the wage or earnings differential between the risky and the safe occupation.

put. Workers naturally prefer higher consumption (output) while avoiding excessive risk. In a competitive equilibrium, we observe that output increases when more workers opt for riskier occupations. However, workers often overlook the fact that this increased sorting into risky jobs causes wages in those occupations to decrease. Consequently, the uninsurable earnings risk in those occupations also decreases due to the multiplicative nature of uninsurable shocks, which represent units of human capital.

When a planner, with the ability to influence workers' occupational choices, considers the impact of the worker mass in a particular occupation on its wage, they tend to encourage more workers to choose risky occupations. This ultimately leads to a rise in aggregate consumption. Even though a larger proportion of workers experience higher uninsurable earnings risk due to the increased number of workers in risky occupations, the individual risk faced by each worker in these occupations is lower compared to the competitive equilibrium. In conclusion, the overall increase in aggregate consumption compensates for the elevated risk in the constrained efficient allocation

As expected, misallocation is more severe the higher the workers' risk aversion. As risk aversion rises, entering the risky industry is less desirable and thus higher risk aversion exacerbates the costs of market incompleteness. We also find that the degree of misallocation is negatively related to the degree of comparative advantage. Independent draws (the extreme case of pure comparative advantage) imply a higher degree of selection because good abilities can only be used in one occupation. When the dependence is low for both abilities there is a higher likelihood that the worker has high ability in at least one occupation. Stronger selection – i.e. the sorting of workers into their higher ability by occupation – implies a better buffer against risk. Therefore, the absence of insurance markets matters less.

For the sake of tractability, margins that can be important for quantifying the impact on output are left out of our analysis. Some of these margins would strengthen the effects of incomplete markets, while others would dampen them. For example, by using a static model we abstract from the dynamic consequences of occupational mobility and savings which are potential insurance mechanisms that alleviate the negative effects of market incompleteness. However, we also abstract from any human capital accumulation prior to entering the labor market that contributes to the formation of worker's skills. For example when a student chooses a major in college. In our framework, the distribution of these skills is exogenous. But market incompleteness can distort worker's skills accumulation decision and through that channel amplify output. This is something that we abstract from. One way to interpret our static model is to think of workers choosing a career (and the single period representing a worker's lifetime). Changes in risk due to, for example, technological progress affect different cohorts of workers at the time they make their career choice (see for example Hobijn, Schoellman, and Vindas (2018)).

Our quantitative analysis focuses on two questions that have received attention in the literature. By making use of the results of Cubas, Silos and Soini (2024), we also compare the competitive equilibrium economy with the first-best. We begin by calibrating the model to US data on earnings by industry. We use estimates of the variance of permanent shocks to earnings by industry, and pick values for the rest of the parameters to match moments from the 2001 wave of the Survey of Income and Program Participation (SIPP). The earnings premium in the data is around 7% (after controlling for observables like education and age) which yields a risk aversion parameter of 2.9. We find that the maximum permanent output loss due exclusively to market incompleteness can be as high as 0.6%. Around 35% of the loss is due to the presence of the pecuniary externality.

We also use our model to quantify the output losses associated with trade reforms. For this purpose, we make use of a number of studies that document a positive relationship between the degree of import penetration and the trade exposure of an industry with the volatility of workers' earnings. We take as given the increase in import penetration of the US manufacturing sector from 1991-2009. This rise in import penetration caused a reallocation of manufacturing workers. In light of our model, the increase in risk due to trade openness makes the tradable sector less attractive for future cohorts of workers. As a result of the increase in risk misallocation rose by 0.1 percentage points of total output. Around 35% of the loss is due to the presence of the pecuniary externality. The corresponding decrease of manufacturing employment predicted by the model is as large as 4 percentage points (a third of that observed in US data).

1.1 Related Literature

Our paper connects several strands of the literature in macroeconomics and labor economics. First, it relates to the macroeconomics literature on misallocation and development. As has been studied in many important papers (see e.g. Hsieh and Klenow (2009), Restuccia and Rogerson (2013), Lagakos and Waugh (2013), Lagakos, Mobarak, and Waugh (2018), Vollrath (2009), Midrigan and Xu (2014), Guner, Ventura, and Yi (2008)) the misallocation of factors of production across firms, sectors or regions within an economy is important to explain cross-country productivity differences. However, with some exceptions (see for example Vollrath (2014) and Hsieh, Hurst, Jones, and Klenow (2019), Buera, Kaboski, and Shin (2011), Bhattacharya, Guner, and Ventura (2013)) the misallocation of human capital has received much less attention. Although the conclusion is that misallocation has large effects, there is no consensus in the sources of misallocation. Researchers have found many specific factors that seem to contribute a small part of the overall effect.

In our case we focus on one particular friction: market incompleteness. On the one hand, this focus allows us to analyze the consequences of a widely studied friction. On the other hand, we abstract from other important barriers to the allocation of workers to occupations and thus our results on misallocation may seem smaller than the ones reported for example in Hsieh, Hurst, Jones, and Klenow (2019). The literature on productivity gains from the re-allocation of capital across productive units focuses on production efficiency because it assumes idiosyncratic productivity risk of firms can be diversified or firms are risk-neutral. We differ from this literature by going one step further: we analyze some aspects of the welfare properties of the economies we study. In particular, by studying the constrained-efficient allocation our paper complements the findings of Davila, Jong, Krusell, and Rull (2012) and Park (2018), but in our case market incompleteness distorts the occupational choice instead of savings or the accumulation of human capital.

Second, the paper builds on literature on human capital and growth in incomplete markets models. In that literature uninsurable earnings risk (risk to human capital) leads to lower human capital investment and economic growth. Examples in this literature are Benabou (2002), Krebs (2003), and Singh (2010). These papers take human capital as a homogeneous asset that increases or decreases a worker's earnings. This paper focuses on the sorting of workers and the occupational distribution that results, when some occupations are riskier than others. The misallocation we focus on is on types of human capital as opposed to human versus physical capital. We emphasize the interaction between the distribution of workers' abilities and market incompleteness in determining the degree of misallocation. Moreover, we provide analytical solutions, which improve the intuition behind our results.

Our theoretical approach uses the insights of Roy (1951) and models workers' occupational choice under uncertainty. Thus, it connects to models of occupational choice used in macroeconomics and labor economics. Examples include Kambourov and Manovskii (2008, 2009), Jovanovic (1979), Miller (1984), Papageorgiou (2014), and Lopes de Melo and Papageorgiou (2016). We focus on the interplay between comparative advantages and risk in shaping worker' occupational choice and thus we complement their findings as well as the ones of Cubas and Silos (2017, 2020), Silos and Smith (2015), Hawkins and Mustre del Rio (2012), Dillon (2016), and Neumuller (2015). We differ from these papers by abstracting from career dynamics so we can obtain closed form solutions and a better characterization of the elements that affect the misallocation of human capital. These simplifications allow us to address aspects of the welfare properties of the economy.

We think our application to trade reforms provides new insights to the literature trying to understand the effects of trade reforms on labor markets. Our framework does not incorporate international trade but it is flexible enough to measure the output losses associated with trade reforms when workers who are exposed to import competition are unable to insure against permanent shocks to their earnings. Thus, our work is also related to the work of Lyon and Waugh (2018), Lee (2020) and Traiberman (2019).

2 Model

Environment In this section we describe the economy an the competitive equilibrium by following Cubas, Silos, and Soini (2024). The economy is static and is populated by a continuum of workers of total mass equal to one. Labor supply is inelastic and their unite of time can be supplied in either of two occupations: the risky occupation (R) and the safe (S).[3] The uncertainty is driven by shocks that alter a worker's ability to perform an occupation; shocks are distributed according to $F_i(y)$ for occupations $i = R, S$. We assume shocks are log-normal and have mean equal to one and $var(log(y_i)) = \sigma_i^2$.

There is a final good produced according to the following CES technology.

$$Y = [\theta N_R^{\nu} + (1-\theta)N_S^{\nu}]^{1/\nu} \tag{1}$$

where N_R and N_S are the aggregate amount of efficiency units of labor in the risky and safe occupations, respectively, $0 < \theta < 1$ governs the share of each occupation in

[3]Focusing on two occupations - one relatively risky and one relatively safe - is done only for simplicity. The framework can be easily generalized to an arbitrary number J of occupations.

total output and ν is the elasticity of substitution between the two occupations.

We assume consumers are born with zero wealth and that they do not save. The absence of wealth is not a restrictive an assumption as it seems: other work (e.g. Cubas and Silos (2017)) has found that savings are useful to weather transitory shocks but not permanent shocks. Since we are focusing on career risk, this risk is permanent from the perspective of the worker. Workers are risk averse and they value consumption levels c according to $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, with $\gamma > 1$.[4]

Workers are endowed with a vector of occupation-specific abilities. These abilities can be thought as skills that are useful in a given occupation (for example, mathematical thinking for an engineer or physical strength for a construction worker). Some abilities may be innate but others can be the result of previously accumulated human capital. Nonetheless, we do not specify the origin of those abilities and we treat them as being predetermined at the time of the occupational choice. Abilities can be correlated across occupations and as a result some workers are likely to excel at several professions. In what follows, the vector of abilities is denoted by $\boldsymbol{X} = (X_R, X_S)$. We model the dependence between the two abilities through a Gumbel copula of two Fréchet random variables:

$$F(x_R, x_S) = Pr(X_R < x_R, X_S < x_S) = \exp\left\{ -\left[\sum_{i \in R,S} (T_i^{\alpha} x_i^{-\alpha})^{1/(1-\rho)} \right]^{(1-\rho)} \right\} \tag{2}$$

where T_i is the scale parameter and α governing the variance. In Cubas, Silos, and Soini (2024) more details. For example, we show the marginal distributions are univariate Fréchet. The parameter ρ (between 0 and 1) governs the degree of dependence across ability levels for of a worker. When it approaches 1 there is perfect dependence between the two ability draws. When it approaches zero, abilities are uncorrelated.

[4]Considering values of γ between 0 and 1 is not a problem for our framework. We restrict γ to be larger than 1 for two reasons: (a) to simplify a slight different notation needed when γ is between 0 and 1, and (b) that the range of values for γ considered in the literature are well above 1.

Occupational Choice The worker chooses consumption levels and the occupation. After drawing a vector of abilities ($\boldsymbol{X} = (x_R, x_S)$) the worker chooses between the risky occupation and the safe occupation.

To formalize the occupational decision given $\boldsymbol{X}$ and the market prices for abilities in each occupation, w_R and w_S, the value of working in occupation i is denoted by $V_i(x_i, w_i)$ and it is equal to:

$$V_i(x_i, w_i) = \max_c \int_{y \in \mathbb{Y}} \frac{c^{1-\gamma}}{1-\gamma} \, dF_i(y) \tag{3}$$

$$\text{subject to } c \leq x_i \, e^y \, w_i$$

To determine the value of working in an occupation the worker needs to know the price of a unit of ability in that occupation, denoted by w_i and the worker's own pre-determined ability x_i. The prices of the skills, w_i, are determined in a competitive equilibrium but taken as given by the worker when choosing an occupation to enter. Once on the job, consumption is constrained by the total amount of ability $x_i \, e^y$ times its price w_i. As shocks y are stochastic with support $\mathbb{Y}$, the value of occupation i is given by the expected utility of consumption.

Among the two alternative careers, the worker picks the one with the highest value.

$$V(\boldsymbol{X}, w_R, w_S) = \max \left\{ V_R(x_R, w_R), V_S(x_R, w_S) \right\} \tag{4}$$

In Cubas, Silos, and Soini (2024) we solve the model which amounts to solve for the proportion of workers in each occupation. It is given by

$$p_R = \frac{T_R^{\frac{\alpha}{(1-\rho)}} \left| \Omega_R(w_R) \right|^{\frac{\alpha}{(1-\rho)(1-\gamma)}}}{\sum\limits_{i \in \{R,S\}} T_i^{\frac{\alpha}{(1-\rho)}} \left| \Omega_i(w_i) \right|^{\frac{\alpha}{(1-\rho)(1-\gamma)}}} \tag{5}$$

where $\Omega_i = \int_{y \in \mathbb{Y}} \frac{(e^y w_i)^{1-\gamma}}{1-\gamma} \, dF_i(y)$.

The proportion of workers, everything else equal, increases with the wage rate.

The proportion of workers also rises if T_R is higher (relative to T_S); a higher T_R raises the comparative advantage for occupation R raising the proportion of workers opting for that occupation.

By knowing the mass of workers performing occupation we then proceed to obtain the amount of efficiency units of labor in each occupation. It is given by the following expression.

$$N_i = p_i \mathbb{E}(\tilde{x}_i) = p_i^{\frac{\alpha-(1-\rho)}{\alpha}} T_i \Gamma\left(1 - \frac{1}{\alpha}\right) \tag{6}$$

where $\mathbb{E}(\tilde{x}_i)$ is the average ability of workers who choose occupation i (i.e. post-sorting).

To obtain this expression we make use of a well-known result: if the marginal distributions of abilities pre-sorting is Fréchet, the post-sorting distribution of abilities is also Fréchet. More specifically, the post-sorting marginal distributions are Fréchet with shape parameter α and scale parameter $T_i p_i^{\frac{-(1-\rho)}{\alpha}}$. These parameters imply a mean ability for occupation i equal to $T_i p_i^{\frac{-(1-\rho)}{\alpha}} \Gamma(1 - \frac{1}{\alpha})$.

Since workers select based on their comparative advantage, in each occupation the average ability after sorting is higher than before sorting. When $\rho = 1$, i.e. when there is perfect dependence of abilities, there is no sorting on relative skills or comparative advantage. In this special case workers are equally skilled (or unskilled) in either occupation. Hence, the distributions of abilities pre- and post-sorting are identical.

Competitive Equilibrium A competitive equilibrium is a pair of efficiency units N_R and N_S, prices w_R and w_S, and output Y_{CE} such that: i) The mass of efficiency units solve the workers' problem, and; ii) wages clear the labor market for each occupation.

Since labor markets are perfectly competitive the wage rate in a given occupation equals the marginal product of employment of that occupation. Using these marginal products and the expression for we can derive a closed form expression for output. In Cubas, Silos, and Soini (2024) we provide details on this derivation but the workers' optimal choice can be summarized by a cutoff productivity level such that the worker to choose the risky occupation if $x_R > \left|\frac{\Omega_S}{\Omega_R}\right|^{\frac{1}{1-\gamma}} x_S$.

The level of output depends on two objects. First, on the shape of the production function, summarized by the share parameter θ and the elasticity of substitution across occupations $1/(1-\nu)$. Second, the level of efficiency units in each occupation. Efficiency units depend on the relative differences in the shape parameter T_i and on the proportion of workers that choose occupation i. This proportion is influenced by γ — the risk aversion coefficient — and its interaction with idiosyncratic risk. The ratio $(E_R/E_S)^{1/(1-\gamma)}$ rises as γ drops, making the riskier occupation relatively more attractive.

To sharpen the intuition we analyze the special case of a Cobb-Douglas technology.

$$Y_{CE} = T_R^{\theta} \left[\frac{\theta E_R^{\frac{1}{1-\gamma}}}{\theta E_R^{\frac{1}{1-\gamma}} + (1-\theta) E_S^{\frac{1}{1-\gamma}}} \right]^{\frac{\theta(\alpha-(1-\rho))}{\alpha}} T_S^{1-\theta} \left[\frac{(1-\theta) E_S^{\frac{1}{1-\gamma}}}{\theta E_R^{\frac{1}{1-\gamma}} + (1-\theta) E_S^{\frac{1}{1-\gamma}}} \right]^{\frac{(1-\theta)(\alpha-(1-\rho))}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right)$$

In the case of Cobb-Douglas it is clear that the mass of workers in the risky occupation rises as risk aversion falls. The ratio $\frac{\theta E_R^{\frac{1}{1-\gamma}}}{\theta E_R^{\frac{1}{1-\gamma}} + (1-\theta) E_S^{\frac{1}{1-\gamma}}}$ rises as γ falls. Everything else constant, less risk aversion raises the fraction of workers in the risky occupation. Efficiency units in the R occupation also rise with the scale parameter T_R. The exponent $\theta(\alpha - (1-\rho))/\alpha$ increases with α for a given θ and ρ. A higher α fattens the upper tail of the abilities distribution, increasing average efficiency and raising output. The role of ρ is also clear from the expression. A higher value implies abilities for a given worker are more correlated, decreasing worker selection, lowering the amount of efficiency units, and therefore lowering output.

3 The Wage Premium and the Compensation for Risk

Differences in risk across occupations imply that workers face a risk-return trade-off in the labor market. This section derives the equilibrium wage differential across occupations and shows how it depends on agents' risk aversion, workers' comparative advantage and the risk spread across occupations.

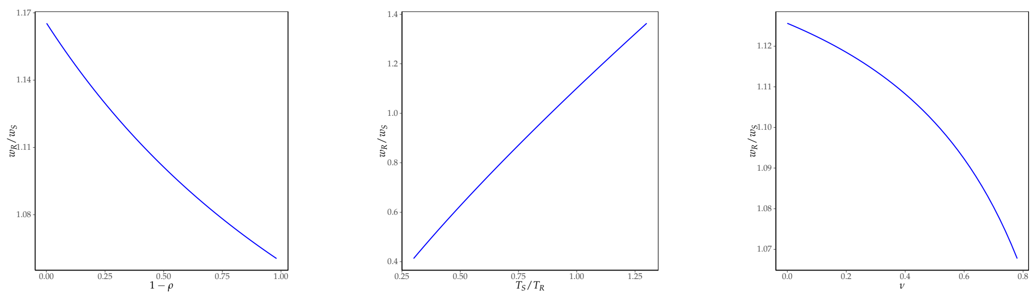
In equilibrium, the ratio of wage rates or prices is the ratio of marginal productivities. Using (1) it can be written as, $WP = \frac{\theta}{1-\theta}\left(\frac{N_R}{N_S}\right)^{\nu-1}$.

Combining (6) and (5) gives the solution for N_R/N_S. Substituting that in, we have that

$$WP = \frac{w_R}{w_S} = \left(\frac{1-\theta}{\theta}\right)^{-\frac{(1-\rho)}{\nu((1-\rho)-\alpha)+\alpha}} \left(\frac{E_S}{E_R}\right)^{\frac{(\alpha-(1-\rho))(1-\nu)}{(\nu((1-\rho)-\alpha)+\alpha)(1-\gamma)}} \left(\frac{T_R}{T_S}\right)^{\frac{\alpha(\nu-1)}{\nu((1-\rho)-\alpha)+\alpha}} \tag{7}$$

The ratio of wages has three components. The first term is related to the shape of the aggregate technology. Everything else constant, wages rise in occupation R if θ falls. The second term, represents the compensation for risk. This premium rises with γ and equals zero when $\gamma = 0$. It also rises with the spread between the variances of the idiosyncratic shocks. The third term represents the influence of the ratio of the means of the distribution of abilities on the ratio of wages. If ability for occupation R is more abundant (T_R is higher) its price drops, everything else constant.

Figure 1



Wage premium and $(1-\rho)$. Wage premium and T_S/T_R. Wage premium and ν.

Notes: The three figures show how wage premium of the risky relative to the safe occupation, varies for different values of three parameters: (a) ρ, (b) T_S/T_R, and (c) ν.

How do the different parameters affect the relative price of the two types of human capital? The answer is shown in Figure 1. We begin by analyzing the changes in the ratio of wage rates w_R/w_S for different values of $(1-\rho)$. This parameter governs the degree of dependence between the abilities of workers, also interpreted as the degree of comparative advantage. When ρ approaches one (zero) it means that the ability draws of a worker are very dependent (non-dependent). In other words, when ρ is close to one, if a worker is good at performing one occupation there is also a high probability of being also good at the other occupation. We can think of ρ approaching one as the limiting case in which there is only one ability to perform both occupations or, just absolute advantage. As it is clear in the picture, the lower ρ is the lower the relative wage rate in occupation R. The reason in this case is simple: when ρ is low then there is more selection in equilibrium. It is always the case that fewer workers choose the risky occupation (because they are risk averse), but the lower the ρ the more selection there is. As a result, workers in the risky occupation are of higher ability, making overall labor in efficiency units larger. Since the technology exhibits diminishing marginal returns to any of the two types of labor, the relative wage in the risky occupation is lower.

The second picture plots the ratio of wages as the ratio T_S/T_R changes. As T_S/T_R increases, the abilities of occupation R are relatively scarce and thus, everything else equal, one unit of human capital of occupation R is relatively more expensive.

The third picture shows the ratio of wage rates for different values of ν, starting with low values – more complementarity across the two occupations in production – up to high values (close to perfect substitutes). The more substitutable occupations are when producing output, the lower the price of one unit of human capital in occupation R relative to occupation S. When occupations are complementary, it is necessary to have workers in both occupations. The only way to attract workers to the risky occupations is a high wage. As the degree of substitution rises, the economy can employ workers in the second occupation without lowering output as much. The need for a high premium is therefore reduced.

The Earnings Premium As opposed to the ratio of wages, the earnings premium is observed in the data. It's defined as the ratio of average earnings across the two occupations:

$$EP = \frac{\frac{w_R N_R}{p_R}}{\frac{w_S N_S}{p_S}} \tag{8}$$

From (6) we can obtain an expression for $\frac{p_S}{p_R}$, i.e.

$$\frac{p_S}{p_R} = \left(\frac{T_R}{T_S}\right)^{\frac{\alpha}{\alpha-(1-\rho)}} \left(\frac{N_S}{N_R}\right)^{\frac{\alpha}{\alpha-(1-\rho)}}. \tag{9}$$

and using the relationship between wages and the labor inputs, and the relationship between the labor inputs and the mass of workers, we obtain,[5]

$$\frac{N_R}{N_S} = \left(\frac{T_S}{T_R}\right)^{\frac{-\alpha}{(1-\rho)}} \left(\frac{\Omega_R}{\Omega_S}\right)^{\frac{\alpha-(1-\rho)}{(1-\rho)(1-\gamma)}}. \tag{10}$$

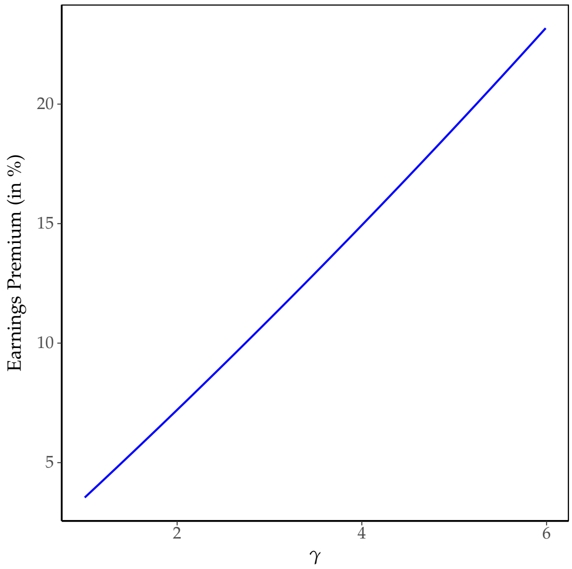
Together with (7) and after some algebra we have that

$$EP = \left(\frac{E_R}{E_S}\right)^{\frac{1}{\gamma-1}}. \tag{11}$$

Interestingly, the earnings premium only depends on the parameters that govern the risk premium, i.e. the relative variance of earnings shocks and the coefficient of risk aversion. As expected, the higher the value of γ the higher the ratio of earnings. This is clearly depicted in 2. Everything else equal the higher the risk aversion, the higher the compensation she/he requires to choose the risky occupation R. For a fixed risk aversion parameter, the higher the volatility of shocks of occupation R relative to S, the higher the compensation for the risk workers face.

[5] The relationship between wages and labor inputs comes from the equality between wages and the marginal product of labor in each occupation. The relationship between the mass of workers and the labor inputs comes from combining (6) and (5).

Figure 2: Risk Aversion and the Earnings Premium



Notes: The figure shows how the earnings premium defined as the average earnings of the risky occupation relative the safe occupation varies with the risk aversion coefficient.

4 Welfare and the Constrained Efficient Allocation

In this section we solve a planning problem in which the planner can assign workers to occupations by distorting their occupational choice, but we constrain the planner by forcing it to respect workers' budget constraints and the market structure. In other words, wages must still equal to marginal products and individual consumption must equal earnings. However, by distorting workers' occupational choice the planner can influence prices, and hence affect welfare. If welfare rises in this constrained planning problem, it means that there are pecuniary externalities that workers do not internalize (see Davila, Jong, Krusell, and Rull (2012) and Park (2018)). This leads to an alternative measure of misallocation: relative to the competitive equilibrium, an alternative allocation raises welfare because prices change but not because idiosyncratic risk is completely eliminated.

As opposed to Davila, Jong, Krusell, and Rull (2012), our model consists of two shocks: the talent shock and the occupational shock. We use the expected utility prior to the realization of these two shocks as the welfare function the planner maximizes.

That is, the planner chooses the allocation that maximizes the expected utility of a new generation of workers before they are born and their talents are realized. Alternatively, one can view that allocation as maximizing expected welfare after the talents have been realized but prior to the realization of the occupational shocks.

We represent the planner's choice of an occupation for a given worker by a cutoff rule ϕ. This rule assigns a worker to the risky occupation if $x_R > \phi x_S$.[6] In the competitive equilibrium a worker's optimal choice was summarized by $x_R > \left|\frac{\Omega_S}{\Omega_R}\right|^{\frac{1}{1-\gamma}} x_S$. This rule implies a fraction of workers in the risky occupation equal to

$$p_R = \frac{1}{1 + \left(\frac{T_S}{T_R}\right)^{\frac{\alpha}{(1-\rho)}} \left(\left|\frac{\Omega_S}{\Omega_R}\right|^{\frac{1}{1-\gamma}}\right)^{\frac{\alpha}{(1-\rho)}}}. \tag{12}$$

This expression is useful because it implies that with the cutoff rule $x_R > \phi x_S$ the fraction of workers in the risky occupation is

$$p_R = \frac{1}{1 + \left(\frac{T_S}{T_R}\right)^{\frac{\alpha}{(1-\rho)}} \phi^{\frac{\alpha}{(1-\rho)}}}^{7}. \tag{13}$$

The constrained-efficient allocation is represented by the value ϕ^* that maximizes the expected utility of workers. In Section A of the Appendix we show that for a given p_R (and thus p_S) the welfare in the economy is given by

$$\left(\Omega_R T_R^{1-\gamma} p_R^{\frac{\alpha-(1-\gamma)(1-\rho)}{\alpha}} + \Omega_S T_S^{1-\gamma} p_S^{\frac{\alpha-(1-\gamma)(1-\rho)}{\alpha}}\right) \Gamma\left(1 + \frac{\gamma - 1}{\alpha}\right) \tag{14}$$

We use this expression to solve for the p_R that maximizes welfare. Then, as in the case of the competitive equilibrium allocation, it is straightforward to obtain the levels of efficiency units of labor in each occupation, wages, and aggregate output. We are

[6] While this cutoff rule distorts the competitive equilibrium, it still respects the sorting of workers based on comparative advantage. For example, if the planner wants to allocate more individuals to the risky occupation, it is optimal to re-allocate individuals with the highest $\frac{x_R}{x_S}$ ratio.

[7] This expression is obtained by just rearranging terms in equation 5.

unable to obtain a simple closed form solution for aggregate output but it is easy to obtain it in a computational analysis. We illustrate the point in our computational experiments and quantitative analysis that follows.

Misallocation The competitive equilibrium is not constrained efficient, which means that by changing the allocation ex ante, the planner can increase expected utility for all individuals.

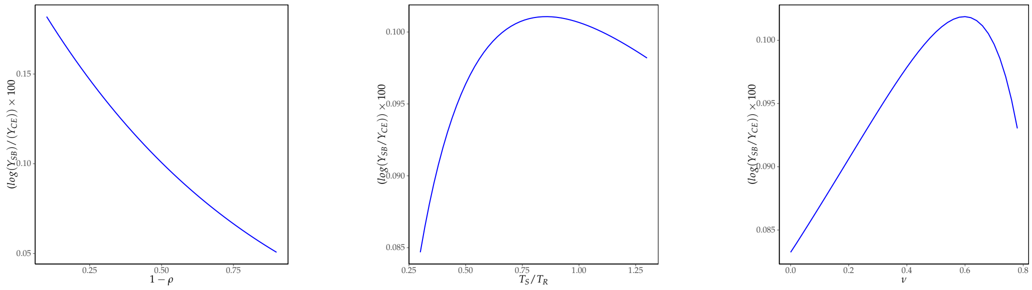
We measure misallocation by comparing the level of output in the second best or constrained-efficient allocation to that in the competitive equilibrium. Specifically, we use the percentage deviation of the competitive equilibrium output (Y_{CE}) from the second-best (Y_{SB}. We do it for different values of ρ, T_S/T_R and ν. In addition, we also present the levels of utility achieved for the average worker in the three types of allocations.

Figure 3 shows how misallocation varies when we change some of the parameters. The first sub-figure shows how misallocation drops as ρ falls ($1-\rho$ rises). The parameter ρ represents the degree of comparative advantage. Thus, when it declines the degree of worker selection rises. This rise in selection rises the mean abilities in the two occupations, providing some degree of insurance to weather idiosyncratic shocks. This implicit insurance increases the fraction of workers in equilibrium to work in the risky occupation.

The second sub-figure shows that as the ratio T_S/T_R rises, misallocation first rises but then declines. When T_S/T_R is low, the mean ability of workers in the risky occupation is high, raising the amount of efficiency units in that occupation and getting the competitive equilibrium closer to the second best. When T_S/T_R is high the mean level of ability in the S occupation is higher, making that occupation relatively more important in determining output. As a result, the level of misallocation drops.

We also analyze how misallocation varies when ν varies. Recall that in the second-best allocation the planner distorts the occupational choice with the goal of affecting

Figure 3



Misallocation and $(1-\rho)$. Misallocation and T_S/T_R. Misallocation and ν.

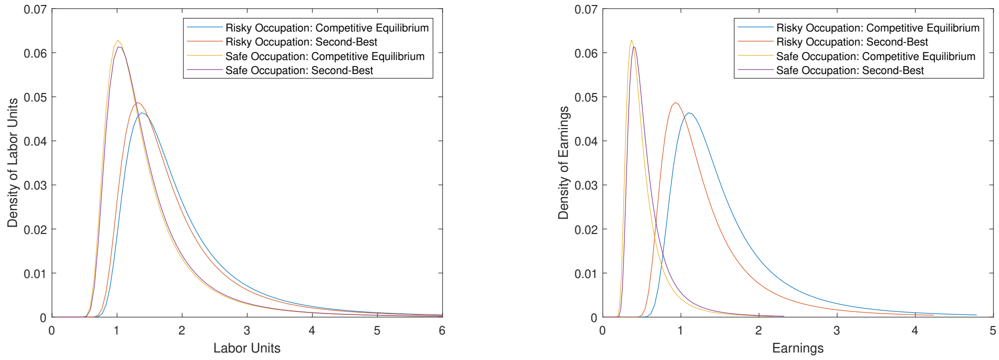
Notes: The three figures show how the degree of misallocation varies for different values of three parameters: (a) ρ, (b) T_S/T_R and (c) ν. Misallocation is measured by the percentage deviation of the competitive equilibrium output (Y_{CE}) from the second best (Y_{SB}).

prices (wages). But when ν increases — the two occupations are substitutable – wages react little when the allocation of workers changes and thus it is difficult for the planner to improve the allocation. However, when ν becomes very large the two occupations become more and more substitutable and thus the differences in the allocations become less important, and hence the second best and the competitive equilibrium allocations become more similar. Therefore, misallocation starts to fall and to approach the competitive equilibrium when ν approaches 1.

Figure 4 shows the distribution of labor units and earnings for competitive equilibrium and the constrained efficient allocation. For the latter, earnings are lower in the risky occupation and higher in the safe occupation. It is worth noting that there are differences in effective labor units as well because sorting changes when the planner changes the cutoff rule. However, most of the differences between the competitive equilibrium and constrained efficient allocation are due to changes in prices.

In Figure 5 we show the proportion of workers in the risky occupation in the competitive equilibrium (p_R^{CE}) and in the second-best (p_R^{SB}) for different values of ρ. As a reference, we also include the proportion in the first-best ($p_R^{FB} = 0.5$). The figure is helpful in understanding the source of the pecuniary externality. It shows that the proportion of workers in the riskier occupation is higher in the second-best than in the competitive equilibrium. Both are lower than the proportion in the first-best

Figure 4

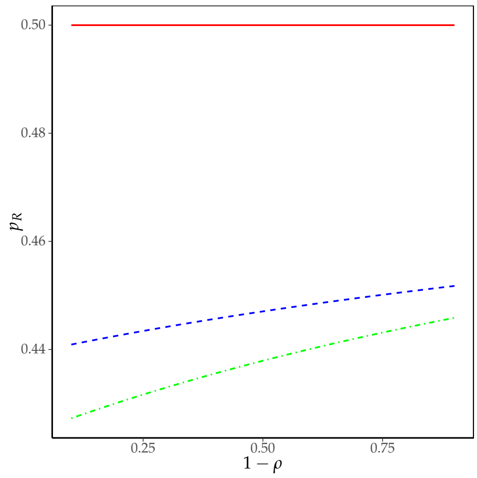


Labor units

Earnings

Notes: The two figures illustrate the post-sorting distribution of labor units and earnings for the competitive equilibrium and constrained efficient allocation. The figure is drawn for a hypothetical economy and does not use the calibrated model values for expositional purposes.

Figure 5: Proportion of Workers in the Risky Occupation



Notes: The figure shows the proportion of workers in the risky occupation in the three different allocations for different values of ρ. The allocations correspond to the competitive equilibrium (p_R^{CE}, dotdashed green), to the first best (p_R^{FB}, solid red) and, to the second best (p_R^{SB}), dashed blue.

allocation. Workers in this economy like consumption but dislike risk. To maximize average welfare the constrained planner wants more workers in the risky occupation because that increases average output, however, it also increases average risk. The competitive equilibrium features too little average consumption for the amount of risk borne. Too many workers in the risky occupation (as in the first-best) imply higher average consumption but also too much average risk. The second-best allocation balances average risk and average consumption. From the competitive equilibrium allocation, all newborns can be made better off by reallocating some of them to work in the riskier occupation. This feature of the model allows us to decompose the distance between the outcome of the competitive equilibrium and the first-best into the part that corresponds to the pecuniary externality and the part associated to risk. In the next section we perform such a decomposition for the US labor market.

The wage premium – the spread between wages in the risky and safe industries – reflects the degree of frictions brought about by incomplete markets, and thus the magnitude of the pecuniary externality. As the amount of labor allocated to the risky occupation rises, the wage premium declines. In the constrained efficient allocation more labor allocated to the risky occupation than in the competitive equilibrium. Hence, the constrained efficient allocation trades off higher expected income from allocating more workers to occupation R, with a lower premium in the wage per efficiency unit of labor.

5 Quantitative Analysis

We use the theoretical model developed in the previous section calibrated to mimic the US economy. We study two cases. We first study misallocation of US workers' due to exposure to industry risk. Second, with the aim of looking at specific sources of risk, we study worker's exposure to risk due to import penetration in their industry of work. In both cases, we decompose the degree of misallocation due to incomplete markets and due to the pecuniary externality.

In our quantitative analysis we provide two notions of misallocation. We do not only compare the allocations of the competitive equilibrium with the constrained planner or second-best allocation but also, the ones of an unconstrained planner problem or first-best allocation. We derive the first best allocations in Cubas, Silos, and Soini (2024). This is an economy with no frictions (markets are complete), where the planner only faces an aggregate resource constraint. In other words, the planner has access to all the tools to complete the market. [8]

5.1 Labor Income Risk and the Misallocation of Workers Across US Industries

Calibrating the model requires parameter values for the variance of the shocks to earnings. The nature of risk faced by workers is important for assessing the welfare consequences of changing social policies. Temporary shocks should not lead to major changes in workers' careers and are easily overcome by a small amount of savings. For that reason, we focus only on permanent (or very persistent) risk that can be associated with, for instance, a depreciation of industry-specific human capital or technological change.

To decompose risk into a permanent component and a transitory component, we follow and use the results of Cubas and Silos (2017) who use the approach of Carroll and Samwick (1997). Using the Survey of Income and Program Participation (SIPP) as the source of earnings data, Cubas and Silos (2017) decompose individual-level earnings in each US industry into a permanent and a transitory component. They estimate the variance of each component, reporting results for a total of 19 industries. We report the details in section B of the Online Appendix.

According to the estimates reported, industries vary greatly in their degree of permanent earnings volatility. We use their estimates and divide industries in two groups, the "risky" and the "safe" sector, according to the variance of the permanent component of earnings. The "risky" group includes Utilities, Finance, Nondurable Goods Manuf., Wholesale Trade, Communication, Retail Trade, Medical Services,

[8] In our framework, there is no leisure choice nor savings. Therefore, the welfare of a newborn who does not know her abilities and shocks is maximized when consumption is maximized or, equivalently, when the economy maximizes output. That is, the planner can deliver the maximum (expected) welfare to this newborn when the economy maximizes output and resources are distributed evenly among all workers.

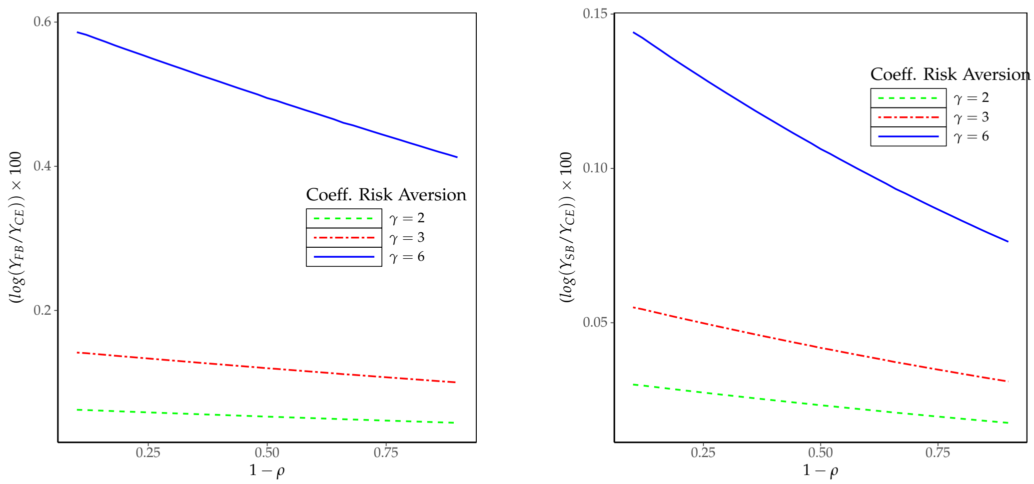
Transportation, Recreation and Entertainment, Construction, Durable Goods Manuf. and Other Services. The "safe" group includes Agriculture and Forestry, Social Services, Government, Hospitals, Business Services, and Personal Services. The first group has a permanent variance of 0.00570 and the second a variance of 0.00399. Because our model is static, we assume a 40-year career for workers and thus multiply each variance by 40. This product represents the variance of the permanent component of earnings over a worker's life-cycle .

We need to calibrate the parameters of the copula, T_R, T_S, α and ρ, in addition to the aggregate technology parameters θ and ν, and the risk aversion parameter γ. Because in our general equilibrium framework mean earnings does not depend on the scale parameters of the Fréchet distribution (T_R and T_S) we fix them at a value of one. To calibrate α we employ the following procedure. Using the 2001 panel of the SIPP we estimate a fixed-effects regression for individual earnings controlling for age and time (the SIPP is a quarterly panel). We interpret the distribution of fixed effects as the distribution of worker productivities prior to experiencing shocks. Consistent with this interpretation we use the standard deviation of fixed effects across workers to calibrate α. Because α is the same for the two abilities distributions, we target the standard deviation of (log) abilities of the safe industry. The standard deviation of workers' fixed effects in the safe industry is 0.345 in the data. We estimate the share parameter θ in the aggregate technology by setting it so that the model delivers a share of workers in the risky industry of 75%, as observed in the data. Finally, to estimate the risk aversion coefficients we derive the expression for the compensation for risk in our environment. In our model, $EP = \left(\frac{E_R}{E_S}\right)^{\frac{1}{\gamma-1}}$. It states that the ratio of average earnings across the two industries depends only on the risk aversion parameter γ and the two standard deviations of the earnings shocks. The earnings premium across the two industries is 6.75%, yielding a risk aversion coefficient of 2.92.

Because we use the standard deviation of earnings to estimate α and the share of workers in the risky industry to estimate θ, we cannot separately estimate ρ. We opt to analyze the model by assuming a range of values for ρ (the minimum is 0.1 and the maximum is 1), recalibrating θ and α for each value of the dependency parameter. Lastly, the parameter ν drives the elasticity of substitution across occupations. The literature lacks a clear reference for an estimate of this elasticity. We opt for a value of ν equal to 1/3 (an elasticity of 1.5). The implied elasticity of that value is halfway between the Cobb-Douglas case (ν equal to 0 or a unit elasticity of substitution) and an elasticity of substitution equal to 3 (or ν equal to 2/3) as used by Hsieh and Klenow (2009). Our chosen value is also close to that estimated by Caunedo, Jaume, and Keller (2021) who use a value of 1.34.

Figure 6 shows the difference between output in the competitive equilibrium and output in the two planner's problems for different values of $(1-\rho)$ and γ.

Figure 6: The Degree of Misallocation Across Industries



First Best Second Best

Notes: The two figures plot the degree of misallocation. In the left panel the degree of misallocation is measured as the percentage deviation of output in a competitive equilibrium from output at the social optimum; i.e. by the percentage deviation of the competitive equilibrium output (Y_{CE}) from the first-best (Y_{FB}). In the right panel the degree of misallocation is measured as the percentage deviation of output in a competitive equilibrium (Y_{CE}) from output in the second-best (Y_{SB}). The horizontal axis represents different values for $(1-\rho)$. The three different lines represent different levels of risk aversion γ.

These results provide a quantitatively plausible range of the level of misallocation. The minimum loss is 0.1% and the maximum loss is around 0.6% of output, permanently. The losses are always lower in the case of the second best allocation because they only represent the cost of the risk (that con't be eliminated by the planner in the second best) but not the cost associated with the pecuniary externality.

In both cases, as ρ decreases the degree of misallocation decreases. Independent draws imply a higher degree of selection because high abilities can only be used in one occupation. When the dependence between abilities is low there is a higher likelihood that the worker has high ability in at least one occupation. The more selection – i.e. the higher average ability by occupation – implies a better buffer against risk and therefore the absence of insurance markets matters less. In addition, for a fixed ρ, the higher the value of the risk aversion parameter γ, the higher the degree of misallocation. As risk aversion rises, entering the risky industry is less desirable. Higher risk aversion exacerbates the costs of market incompleteness.

5.2 Risk, Import Penetration and the Misallocation of Workers

In our previous analysis we are silent about the sources of differences in the variance of permanent risk across industries. However, there is a growing number of studies that relate the degree of import penetration and trade exposure of an industry with the volatility of workers' earnings. An important paper in this literature is Krishna and Senses (2014) who document that a 10% increase in import penetration in an industry is associated with a 23% increase in the variance of permanent shocks to labor earnings.

As a consequence, as documented by a large body of literature on labor and trade, the increase in import competition has dramatically changed US labor markets. An important aspect is the increased importance of China as a competitive producer of manufactures after it entered the World Trade Organization. These authors document that the increase in import penetration of manufactures in the US accounts for a total loss of 12% of manufacturing employment in the United States.[9]

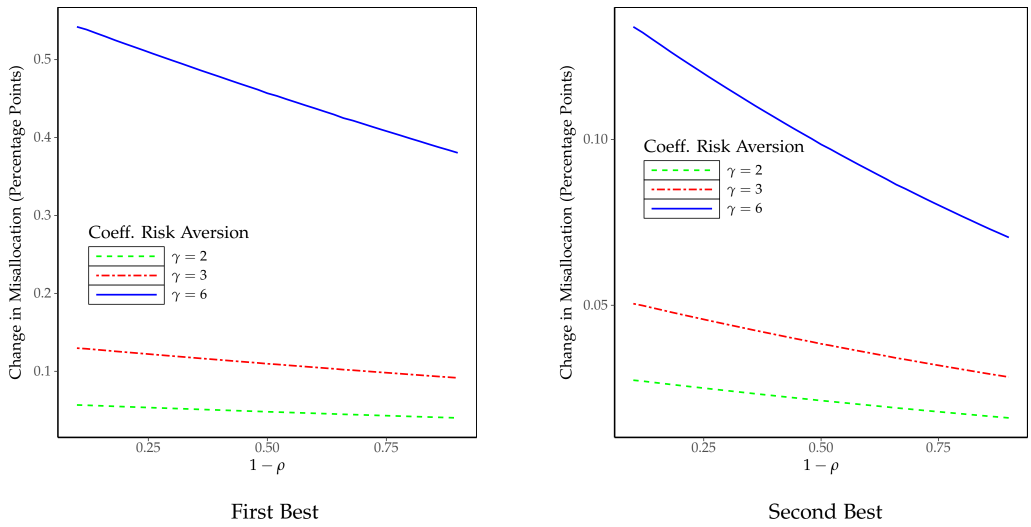
[9] Acemoglu, Autor, Dorn, Hanson, and Price (2016) report employment losses of about 2.2 million.

We use our framework to connect these two strands of the literature. We examine the output costs derived from the increase in import penetration in the tradable sector. In light of our model, everything else equal, the new cohort of risk averse workers tries to avoid the tradeable sector since the increase in risk due to trade openness makes the sector less attractive. We use our previous calibration but we now divide industries in two groups: "tradables" and "non-tradables". The tradable group comprises Durable Goods Manufacturing, Non-Durable Goods Manufacturing and Agricultural and Forestry. All other industries are included in non-tradables. The variances of the permanent shocks to earnings are 0.0061 and 0.0050 for the tradable sector and non-tradable sector, respectively. We interpret the allocations of our model with this parameterization as an initial steady state and entertain a trade reform to measure the change in the degree of misallocation. For this purpose, we use the estimates of Acemoglu, Autor, Dorn, Hanson, and Price (2016) who document an increase in the import penetration in the manufacturing sector of 7%. In addition, according to estimates of Krishna and Senses (2014), an increase of import penetration of 7%, corresponds to an increase in the variance of the permanent shock to labor earnings of the tradable sector of 16.1%. Thus, according to our estimates, the variance of the tradable sector would be 0.0070. Ceteris paribus, in the new equilibrium with a riskier tradable sector the model predicts an increase in the degree of misallocation and a decrease in the number of workers in the tradable sector.

Figure 7 shows the change in misallocation for different values of ρ and γ. We measure misallocation the same way as before: the percentage change of the competitive equilibrium output from the first-best and the second-best. The figure plots the change in misallocation as trade opens. For example, if misallocation is 1% pre-trade and 1.5% post-trade, the change in misallocation is half a percentage point. For a given value of ρ and γ, there is an increase in misallocation following the trade reform. After the increase in trade openness the tradable industry is even riskier than

Manufacturing employment in January of 1999 was about 17 million workers.

Figure 7: Import Penetration and Misallocation



Notes: The two figures plot the degree of misallocation. In the left panel the degree of misallocation is measured as the percentage deviation of output in a competitive equilibrium from output at the social optimum; i.e. by the percentage deviation of the competitive equilibrium output (Y_{CE}) from the first-best (Y_{FB}). In the right panel the degree of misallocation is measured as the percentage deviation of output in a competitive equilibrium (Y_{CE}) from output in the second-best (Y_{SB}). The horizontal axis represents different values for $(1-\rho)$. The three different lines represent different levels of risk aversion γ.

the non-tradable industry. As a result, less workers enter the tradable sector, resulting in an allocation that is farther away from the planner's allocation than was pre-trade allocation. The magnitude of this increase in misallocation depends upon the values of ρ and γ. The figures show the same pattern as in the previous quantitative application: misallocation increases as risk aversion and/or the degree of dependence of abilities increases. As the picture shows, the increase in misallocation can plausibly be as large as 0.55 percentage points and it is larger when the competitive equilibrium allocation is compared with the first-best allocation than when compared to the second-best allocation.

To calculate how much of the difference in output is due to the presence of a pecuniary externality, we proceed in the following way. We calculate the difference in output between the first best and the second best as a fraction of the difference in output between the first best and the competitive equilibrium. Subtracting that ratio from 100% gives us the fraction of output losses due to the pecuniary externality.

For the range of parameter values considered, the average value of that calculation is about 35%.

6 Conclusions

How does the lack of insurance markets to insure against worker's permanent earnings shocks affect their occupational or industry choice and the allocation of human capital in an economy? What are the consequences for aggregate productivity? We have answered these questions by developing a Roy model of occupational choice with incomplete markets. Risk averse workers choose an occupation based on the occupation-specific risk they face and on their comparative and absolute advantages. The tractability of the Frechet distribution allows for a closed-form solution of the competitive equilibrium allocation. In a competitive equilibrium, human capital is misallocated because workers avoid risky industries. The social planner allocates more workers to risky industries. The higher the risk aversion and the lower the degree of comparative advantage, the larger the misallocation. We perform two quantitative exercises to measure the size of misallocation when comparing the competitive equilibrium with the first-best and second-best allocations. We estimate that only this friction can generate a permanent output loss of 0.6%.

The paper abstracts from the welfare implications of different tax and transfer schemes that would simultaneously improve welfare and output. The reason is that the analytic tractability is lost unless the tax and transfer is too blunt (e.g. a fixed tax for safe occupations and a fixed subsidy to risky occupations.) and unlikely to improve welfare. More realistic and flexible tax and transfer schemes can only be analyzed through numerical solutions. Nonetheless, it is an important question that we hope to tackle in future research.

We think this paper offers a new perspective for understanding the link between risk in labor markets and the aggregate levels of human capital. We focus on the interplay between abilities and risk. We abstract from many aspects of the labor market and the career choice of the individuals. For instance, we take earnings volatility as exogenous and we do not consider heterogeneity in risk aversion. For the sake of tractability and to obtain analytical expressions we also abstract from the career dynamics and the role that savings play in shaping the occupational choice. We also abstract from many barriers that surely affect the occupational choice and mobility of workers and that may interact with the lack of insurance. From this perspective, we think our measured misallocation can be a lower bound in our quantitative exercises. We hope our findings encourage future research that relaxes these assumptions.

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Appendix

A The Second-Best Allocation

The planner maximizes welfare by choosing a cutoff rule $\phi > 0$, so that a worker goes to R if $x_R > \phi x_S$.

Note that according to Proposition 2.1, a choice rule $x_R > \left|\frac{\Omega_S}{\Omega_R}\right|^{\frac{1}{1-\gamma}} x_S$ leads to $p_R = \frac{1}{1+\left(\frac{T_S}{T_R}\right)^{\frac{\alpha}{(1-\rho)}}\left(\left|\frac{\Omega_S}{\Omega_R}\right|^{\frac{1}{1-\gamma}}\right)^{\frac{\alpha}{(1-\rho)}}}$. With the same reasoning, a choice rule $x_R > \phi x_S$ leads to

$$p_R = \frac{1}{1+\left(\frac{T_S}{T_R}\right)^{\frac{\alpha}{(1-\rho)}}\phi^{\frac{\alpha}{(1-\rho)}}} \tag{15}$$

We also know that $p_S = 1 - p_R$ and $N_i = p_i^{\frac{\alpha-1}{\alpha}}\Gamma\left(1-\frac{1}{\alpha}\right)$. Hence the choice of ϕ pins down a unique labor allocation which in turn pins down unique wages.

Proposition A.1 *When the planner picks a particular ϕ, the welfare in the economy is given by*

$$\left(\frac{T_R^{\frac{\alpha}{1-\rho}}\Omega_R}{\left(T_R^{\frac{\alpha}{1-\rho}}+(T_S\phi)^{\frac{\alpha}{1-\rho}}\right)^{\frac{\alpha-(1-\gamma)(1-\rho)}{\alpha}}}+\frac{T_S^{\frac{\alpha}{1-\rho}}\Omega_S}{\left((T_R\phi^{-1})^{\frac{\alpha}{1-\rho}}+T_S^{\frac{\alpha}{1-\rho}}\right)^{\frac{\alpha-(1-\gamma)(1-\rho)}{\alpha}}}\right)\Gamma\left(1+\frac{\gamma-1}{\alpha}\right) \tag{16}$$

Furthermore, wages in Ω_R and Ω_S can be computed using (15) *together with $p_S = 1 - p_R$, $N_i = p_i^{\frac{\alpha-1}{\alpha}}\Gamma\left(1-\frac{1}{\alpha}\right)$ and the wage equation resulting from the CES production function.*

Proof For a given cutoff rule ϕ, the expected utility of those going to R is given by

$$\begin{aligned} E\left(x_R^{1-\gamma}\Omega_R, x_R > \phi x_S\right) &= \int_0^\infty \int_0^{\frac{x_R}{\phi}} x_R^{1-\gamma}\Omega_R f_{x_R,x_S}(x_R,x_S)dx_S dx_R \\ &= \Omega_R \int_0^\infty \int_0^{\frac{x_R}{\phi}} x_R^{1-\gamma} f_{x_R,x_S}(x_R,x_S)dx_S dx_R \end{aligned} \tag{17}$$

Here Ω_R is a function of wages which depend only on ϕ so Ω_R can be moved outside the integral. As discussed in Appendix A, the joint density f_{x_R,x_S} can be expressed as

$$\begin{aligned} f_{x_R,x_S}(x_R,x_S) &= \frac{d^2}{dx_R dx_S} F_{x_R,x_S}(x_R,x_S) \\ &= \frac{d}{dx_S} F_{x_R,x_S}(x_R,x_S) \left(T_R^{\frac{\alpha}{1-\rho}} x_R^{\frac{-\alpha}{1-\rho}} + T_S^{\frac{\alpha}{1-\rho}} x_S^{\frac{-\alpha}{1-\rho}} \right)^{-\rho} \alpha T_R^{\frac{\alpha}{1-\rho}} x_R^{\frac{-\alpha}{1-\rho}-1} \end{aligned} \tag{18}$$

Plugging this into (17), the inner integral cancels out and we can substitute in the boundary of the integral: $x_S = x_R/\phi$. As a result, the expected utility becomes:

$$\begin{aligned} E\left(x_R^{1-\gamma}\Omega_R, x_R > \phi x_S \right) &= \Omega_R \int_0^\infty x_R^{1-\gamma} \exp\left(- \left(T_R^{\frac{\alpha}{1-\rho}} x_R^{\frac{-\alpha}{1-\rho}} + T_S^{\frac{\alpha}{1-\rho}} \left(\frac{x_R}{\phi} \right)^{\frac{-\alpha}{1-\rho}} \right)^{1-\rho} \right) \\ &\quad \times \left(T_R^{\frac{\alpha}{1-\rho}} x_R^{\frac{-\alpha}{1-\rho}} + T_S^{\frac{\alpha}{1-\rho}} \left(\frac{x_R}{\phi} \right)^{\frac{-\alpha}{1-\rho}} \right)^{-\rho} \alpha T_R^{\frac{\alpha}{1-\rho}} x_R^{\frac{-\alpha}{1-\rho}-1} dx_R \\ &= \Omega_R \int_0^\infty x_R^{1-\gamma} \exp\left(- \tilde{T} x_R^{-\alpha} \right) \tilde{T}^{\frac{-\rho}{1-\rho}} x_R^{\frac{\alpha\rho}{1-\rho}} \alpha T_R^{\frac{\alpha}{1-\rho}} x_R^{\frac{-\alpha}{1-\rho}-1} dx_R \end{aligned} \tag{19}$$

where $\tilde{T} = \left(T_R^{\frac{\alpha}{1-\rho}} + (T_S\phi)^{\frac{\alpha}{1-\rho}} \right)^{1-\rho}$.

Defining $x = \tilde{T} x_R^{-\alpha}$, we get $dx = -\tilde{T}\alpha x_R^{-\alpha-1} dx_R$ and $x_R = \left(\frac{x}{\tilde{T}} \right)^{-1/\alpha}$.

Using a change of variables in the integral leads to

$$\begin{aligned} &\Omega_R \int_0^\infty \left(\frac{x}{\tilde{T}} \right)^{-(1-\gamma)/\alpha} \exp\left(-x \right) \tilde{T}^{\frac{-\rho}{1-\rho}-1} T_R^{\frac{\alpha}{1-\rho}} dx \\ =&\Omega_R T_R^{\frac{\alpha}{1-\rho}} \tilde{T}^{\frac{(1-\gamma)(1-\rho)-\alpha}{\alpha(1-\rho)}} \Gamma\left(1 + \frac{\gamma-1}{\alpha} \right) = \frac{T_R^{\frac{\alpha}{1-\rho}} \Omega_R}{\left(T_R^{\frac{\alpha}{1-\rho}} + (T_S\phi)^{\frac{\alpha}{1-\rho}} \right)^{\frac{\alpha-(1-\gamma)(1-\rho)}{\alpha}}} \Gamma\left(1 + \frac{\gamma-1}{\alpha} \right) \end{aligned} \tag{20}$$

This is the average utility of a worker who goes to occupation R. Since a worker goes to S if $x_R\phi^{-1} < x_S$, we can get the average utility of a worker who goes to S by swapping the labels R and S in (20) and replacing ϕ with ϕ^{-1}. The average utility of a worker who goes to S is therefore given by

$$\frac{T_S^{\frac{\alpha}{1-\rho}}\Omega_S}{\left((T_R\phi^{-1})^{\frac{\alpha}{1-\rho}}+T_S^{\frac{\alpha}{1-\rho}}\right)^{\frac{\alpha-(1-\gamma)(1-\rho)}{\alpha}}}\Gamma\left(1+\frac{\gamma-1}{\alpha}\right) \tag{21}$$

Total average utility in the economy is the sum of (20) and (21):

Therefore, the expected welfare for given $\phi \in \mathbb{R}_+$ is given by

$$\left(\Omega_R\frac{T_R^{\frac{\alpha}{1-\rho}}}{\left(T_R^{\frac{\alpha}{1-\rho}}+(T_S\phi)^{\frac{\alpha}{1-\rho}}\right)^{\frac{\alpha-(1-\gamma)(1-\rho)}{\alpha}}}+\Omega_S\frac{T_S^{\frac{\alpha}{1-\rho}}}{\left((T_R\phi^{-1})^{\frac{\alpha}{1-\rho}}+T_S^{\frac{\alpha}{1-\rho}}\right)^{\frac{\alpha-(1-\gamma)(1-\rho)}{\alpha}}}\right)\Gamma\left(1+\frac{\gamma-1}{\alpha}\right) \tag{22}$$

which is the same as

$$\left(\Omega_R\frac{T_R^{1-\gamma}\left(T_R^{\frac{\alpha}{1-\rho}}\right)^{\frac{\alpha-(1-\gamma)(1-\rho)}{\alpha}}}{\left(T_R^{\frac{\alpha}{1-\rho}}+(T_S\phi)^{\frac{\alpha}{1-\rho}}\right)^{\frac{\alpha-(1-\gamma)(1-\rho)}{\alpha}}}+\Omega_S\frac{T_S^{1-\gamma}\left(T_S\phi\right)^{\frac{\alpha}{1-\rho}\frac{\alpha-(1-\gamma)(1-\rho)}{\alpha}}}{\left(T_R^{\frac{\alpha}{1-\rho}}+(T_S\phi)^{\frac{\alpha}{1-\rho}}\right)^{\frac{\alpha-(1-\gamma)(1-\rho)}{\alpha}}}\right)\Gamma\left(1+\frac{\gamma-1}{\alpha}\right) \tag{23}$$

But the cutoff rules for given ϕ in terms of p_R are

$$p_R=\frac{T_R^{\frac{\alpha}{(1-\rho)}}}{T_R^{\frac{\alpha}{(1-\rho)}}+\left(T_S\phi\right)^{\frac{\alpha}{(1-\rho)}}}. \tag{24}$$

and

$$p_S=\frac{\left(T_S\phi\right)^{\frac{\alpha}{(1-\rho)}}}{T_R^{\frac{\alpha}{(1-\rho)}}+\left(T_S\phi\right)^{\frac{\alpha}{(1-\rho)}}}. \tag{25}$$

Substituting these back to the previous equation gives a simpler expression for the expected welfare for any choice of p_R and p_S:

$$\left(\Omega_R T_R^{1-\gamma} p_R^{\frac{\alpha-(1-\gamma)(1-\rho)}{\alpha}} + \Omega_S T_S^{1-\gamma} p_S^{\frac{\alpha-(1-\gamma)(1-\rho)}{\alpha}}\right) \Gamma\left(1+\frac{\gamma-1}{\alpha}\right) \tag{26}$$

B Estimation of Income Risk for US Industries

An important input in our calibration are the estimation of industry-level wage regressions and the variances of the shocks to the earnings workers face in each industry. We follow and use the results of Cubas and Silos (2017). In this section, we briefly describe the dataset, the estimation method used to estimate the labor earnings processes and the assumed properties of the shocks faced by workers in different industries.

The definition of labor earnings is rather broad (but consistent with previous studies). It captures the variability in wage rates but also changes in earnings due to changes in the number of hours worked or changes in employment status. We focus on individuals who never change industries as this is most consistent with the quantitative framework we use.

We use the Survey of Income and Program Participation (SIPP). The SIPP is constructed by the U.S. Census Bureau and takes the form a series of continuous panels that follow a national sample of households. We use the 1996, 2001, and 2004 panels obtained from the Center for Economic and Policy Research, CEPR (2014). The SIPP sample is considerably larger than that of the PSID and thus, it allows us to have 19 industries with a significant number of workers in each of them.

We use quarterly data constructed from the monthly data provided in SIPP. The sample is composed by individuals between 22 and 66 years of age with at least 10 consecutive quarters of responses. We eliminate those who are self-employed and those out of the labor force.

The first step in our analysis computes earnings variability at the individual level with a regression approach used extensively in the literature, see for example, Carroll and Samwick (1997). We proceed by estimating a fixed effects model for each industry j in our sample. Given a panel of N individuals for whom we measure earnings (and other variables) over a period of time T, we assume that (log) earnings for individual i in industry j at time t, y_{ijt} can be modeled as

$$y_{ijt} = \alpha_{ij} + \boldsymbol{\beta}_j \boldsymbol{X}_{ijt} + u_{ijt} \tag{27}$$

The vector $\boldsymbol{X}$ includes several variables that help predict changes in the level of log earnings. Specifically, we include age, sex, ethnicity, years of schooling, an occupational dummy, and time dummies.

We estimate equation (27) for all individuals in a given industry. Repeating this procedure for all industries yields estimates $\{\hat{\alpha}_{ij}, \hat{\boldsymbol{\beta}}_j\}_{j=1}^{19}$.

To account for this difference in the nature of risk, we enrich our empirical analysis by allowing the error term to be decomposed into a permanent component and a transitory component. We follow Carroll and Samwick (1997) and Low, Meghir, and Pistaferri (2010), among others, by assuming that the residual is equal to the addition of a permanent and a transitory component. In addition, given we use quarterly data we enrich our analysis by allowing for the possibility of no occurrence of the shocks in every quarter. Moreover, we allow the probability of the occurrence of the shocks to be industry-specific.

Thus, we assume that

$$u_{ijt} = \eta_{ijt} + \omega_{ijt}, \tag{28}$$

where η_{ijt} is the transitory component and ω_{ijt}, the permanent component which is a random walk, that is,

$$\omega_{ijt} = \omega_{ij,t-1} + \epsilon_{ijt}. \tag{29}$$

As mentioned above, we further assume that

$$\eta_{ijt} = \begin{cases} 0 & \text{with probability} \quad \phi_j \\ \tilde{\eta}_{ijt} & \text{with probability} \quad 1-\phi_j \end{cases} \tag{30}$$

with $\tilde{\eta}_{ijt}$ distributed i.i.d. $N(0, \sigma^2_{\tilde{\eta},j})$; and

$$\epsilon_{ijt} = \begin{cases} 0 & \text{with probability} \quad \lambda_j \\ \tilde{\epsilon}_{ijt} & \text{with probability} \quad 1-\lambda_j \end{cases} \tag{31}$$

with $\tilde{\epsilon}_{ijt}$ distributed i.i.d. $N(0, \sigma^2_{\tilde{\epsilon},j})$.

The estimation of equation (27), we obtain $\{\{\hat{u}_{ijt}\}_{i=1}^{N_j}\}_{t=1}^{T}$. Using those and for each industry j, we estimate the vector of parameters $\{\sigma^2_{\tilde{\epsilon},j}, \sigma^2_{\tilde{\eta},j}, \lambda_j, \phi_j\}$ by the method of moments. We follow an identification procedure similar to the one proposed by Low, Meghir, and Pistaferri (2010) in which the moments to match are $E[(\Delta u_{ijt})^2]$, $E[(\Delta u_{ijt})^4]$, $E[\Delta u_{ijt}\Delta u_{ijt-1}]$ and $E[(\Delta u_{ijt})^2(\Delta u_{ijt-1})^2]$. The theoretical expressions for these moments are functions of the vector of parameters and they are given by the following equations.

$$E[(\Delta u_{ijt})^2] = 2(1-\phi)\sigma^2_{\tilde{\eta},j} + (1-\lambda)\sigma^2_{\tilde{\epsilon},j} \tag{32}$$

$$E[(\Delta u_{ijt})^4] = 6(1-\phi)^2\sigma^4_{\tilde{\eta},j} + 12(1-\phi)(1-\lambda)\sigma^2_{\tilde{\eta},j}\sigma^2_{\tilde{\epsilon},j} + 6(1-\phi)\sigma^4_{\tilde{\eta},j} + 3(1-\lambda)\sigma^4_{\tilde{\epsilon},j} \tag{33}$$

$$E[\Delta u_{ijt}\Delta u_{ijt-1}] = -(1-\phi)\sigma^2_{\tilde{\eta},j} \tag{34}$$

and

$$\begin{aligned} E[(\Delta u_{ijt})^2(\Delta u_{ijt-1})^2] = 3(1-\phi)^2\sigma^4_{\tilde{\eta},j} + 4(1-\phi)(1-\lambda)\sigma^2_{\tilde{\eta},j}\sigma^2_{\tilde{\epsilon},j} + (1-\lambda)^2\sigma^4_{\tilde{\epsilon},j} + \\ (1-\phi)\sigma^4_{\tilde{\eta},j}. \end{aligned} \tag{35}$$

To estimate the variances of the two innovations, we proceed as follows. For a sample of workers in a given industry j, we estimate $E(\widehat{\Delta u_{ijt}\Delta u_{ijt}})$, $E(\widehat{\Delta u_{ijt}\Delta u_{ijt-1}})$, $E(\widehat{[\Delta u_{ijt}\Delta u_{ijt}]^2})$ and $E(\widehat{[\Delta u_{ijt}\Delta u_{ijt-1}]^2})$ using the sample analogs. Solving the system comprised of the previous four equations, we obtain $\widehat{\sigma}^2_{\tilde{\epsilon},j}$, $\widehat{\sigma}^2_{\tilde{\eta},j}$, $\widehat{\lambda}_j$ and $\widehat{\phi}_j$. As a result, the estimates of the variances of the permanent and transitory shocks are $\widehat{\sigma}^2_{\epsilon_j} = (1 - \widehat{\lambda}_j)\widehat{\sigma}^2_{\tilde{\epsilon}_j}$ and $\widehat{\sigma}^2_{\eta_j} = (1 - \widehat{\phi}_j)\widehat{\sigma}^2_{\tilde{\eta}_j}$, respectively.

The tables in Cubas and Silos (2017) contain all the moments we use to calibrate our model.